

An
Experimental Study of Certain
Factors Affecting Transfer of
Training in Arithmetic

BY
JAMES ROBERT OVERMAN

A Dissertation Submitted in
Partial Fulfillment of the Requirements for
the Degree of Doctor of Philosophy,
in the University of Michigan

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An Experimental Study of Certain Factors Affecting Transfer of Training in Arithmetic

CHAPTER I

INTRODUCTION

I. SUMMARY OF THE LITERATURE ON TRANSFER OF TRAINING

Any new experiment concerning transfer of training must of necessity take into consideration the results of previous studies of the same question. The brief summary here given of experiments and of conclusions drawn from these experiments by their authors and others, is intended to show the changes that have occurred in the attitude of psychologists toward this question within the last thirty-five years, and to make clear the present status of the problem.

1. *Experiments Showing Limitations of Transfer.*—As early as 1896 Jastrow¹ found that Hermann and Kellar, experts at sleight-of-hand, were somewhat below the average of college students in the speed of reactions with discriminations and choice, and in laboratory tests of tactual and muscular reaction. Raif,² in 1900, found

¹ Jastrow, J. "Psychological Notes upon Sleight of Hand Experts," *Science*, N. S., III, 685-689.

² Raif, O. "Ueber Fingerfertigkeit beim Clavierspiel," *Zeitschrift für Psychologie*, XXIV, 352-356.

that expert pianists could not tap any more rapidly than ordinary persons of the same degree of general ability. In 1901, Thorndike and Woodworth¹ published three papers describing the limited extent to which the effects of training in estimating areas, lengths, and weights of a certain shape and size spread to estimating areas, lengths, and weights similar in shape but different in size, different in shape but similar in size, and different in both shape and size.

These early experiments were followed by many others which tended to show that training in one mental function carried over in very small degree to other mental functions. A few of these experiments were in the field of arithmetic. In 1910, Winch² tried to determine whether improvement in accuracy of numerical computation would result in improvement in arithmetical reasoning. Pupils in four schools were divided on the basis of preliminary tests in working arithmetical problems into two equal groups. One of these groups was practiced in working rule sums. After a certain number of days further sets of problems were used to test both groups in arithmetical reasoning. On the reasoning tests marks were given with reference to the process only, no attention being paid to the right answer. No other arithmetical work was done during the time the experiment lasted. It was found that some sections of the control group did better than the practiced group on the final test, whereas others did not do as well. The

¹ Thorndike, E. L., and Woodworth, R. S. "The Influence of Improvement in One Mental Function upon the Efficiency of Other Functions," *Psychological Review*, VIII, 247-261, 384-395, 553-564.

² Winch, W. H. "Accuracy in School Children. Does Improvement in Numerical Accuracy 'Transfer'?", *Journal of Educational Psychology*, I, 557-589.

differences were small in all cases. The author concludes that it seems possible to improve the accuracy of numerical computation without any certainty that we shall thereby improve the accuracy of arithmetical reasoning.

In a further experiment, Winch¹ reaches similar conclusions. A class of 72 boys, with an average age slightly over ten years, was the subject of this experiment. The procedure was the same as in the previous experiment, except the pupils were not asked to solve the problems given in the problem tests, but simply to tell how to solve them. The practiced group showed an average improvement of 40 per cent as the result of ten days practice on fundamentals. The non-practiced group obtained an average score of 42.2 on the preliminary test and 45.7 on the final tests. The practiced group made scores of 42.0 and 45.3. The author concludes that the great improvement in accuracy of arithmetical computation seems to have produced no improvement whatever in the accuracy of arithmetical reasoning.

A number of different experiments have been carried out to determine the influence of improvement in memorizing one type of material on memorizing material of a different type. In 1911, Sleight² reported two elaborate series of experiments of this type. The test series consisted of ten tests in memorizing (1) Location of points in circles, (2) Dates, (3) Nonsense syllables, (4) Poetry, (5) Prose, (6) Prose substance, (7) Locating capes, bays, etc. on a map, (8) Dictated prose, (9) Letters, (10) Names. The training series were of three sorts. Part of the subjects were practiced on learning poetry by heart,

¹ Winch, W. H. "Further Work on Numerical Accuracy in School Children. Does Improvement in Numerical Accuracy Transfer?," *Journal of Educational Psychology*, II, 262-271.

² Sleight, W. G. "Memory and Formal Training," *British Journal of Psychology*, IV, 386-457.

part on memorizing mathematical tables, and part on remembering and writing out the gist of prose selections read to them twice. In neither of the two experiments performed with these tests did the practiced groups show any substantial superiority on the test series over the unpracticed groups. In general there was little or no spread of improvement.

2. *Effect of These Experiments.*—As a result of experiments, such as those described above, which either showed no transfer or else much less than would have been previously expected, there was a general tendency on the part of psychologists and educators to swing from a belief in general transfer to a denial of any transfer. This attitude is well expressed in the following statement by Knight and Setzafandt.

Characteristic of the whole-or-none attitude of American thinking, when once our faith in general transfer was taken from us by competent research, notably the work of Thorndike, we went over, for some time, to the theory that there was no transfer whatever. We learned exactly what we practiced and nothing more. Addition of whole numbers was a skill developed by adding whole numbers; subtraction of whole numbers was a skill derived from subtracting whole numbers. More specifically, adding 5 and 6 taught us how to add 5 and 6 and nothing else.¹

3. *The Reaction.*—Most psychologists have realized for some time that the lack of transference has in the past often been exaggerated and that this exaggeration is as dangerous as the former belief in complete and general transfer. Thorndike shows that he is aware of this danger when he writes:

¹ Knight, F. B., and Setzafandt, A. O. H. "Transfer within a Narrow Mental Function," *Elementary School Journal*, XXIV (June, 1924), 780.

The real question is not, "Does improvement in one function alter others?" but, "To what extent, and how, does it?"¹

The notions of mental machinery which, being improved for one sort of data, held the improvement equally for all sorts; of magic powers which, being trained by exercise of one sort to a high efficiency, held that efficiency whatever they might be exercised upon; and of the mind as a reservoir for potential energy which could be filled by any one activity and drawn on for any other—have now disappeared from expert writings on psychology. A survey of experimental results is now needed perhaps as much to prevent the opposite superstition; for, apparently, some careless thinkers have rushed from the belief in totally general training to the belief that training is totally specialized.²

The same attitude is taken by Stratton when he writes "The experiments in clear support of the doctrine—that you train merely what you train—are few; most experiments contradict it."³ Again Pillsbury writes "A few men hold and more have held that the result indicates that there is no transfer of training whatsoever."⁴ Finally Knight and Setzafandt sum up the matter when they write "Absence of *any* transfer is as false and probably as harmful a notion to use in curriculum construction as its antithesis, complete and magical transfer through the powers of the mind."⁵

4. *Experiments Showing Transfer*.—Experimental data are not lacking to show that transfer does take place in considerable amounts, at least under certain conditions. Starch,⁶ in 1911, reported an experiment on

¹ Thorndike, E. L. *Educational Psychology*, II, 358. New York: Teachers College, Columbia University, 1913.

² Thorndike, E. L. *op. cit.*, 364–365.

³ Stratton, G. M. *Developing Mental Power*, 12. Boston: Houghton Mifflin Company, 1922.

⁴ Pillsbury, W. B. *Education as the Psychologist Sees It*, 300. New York: Macmillan, 1925.

⁵ Knight, F. B., and Setzafandt, A. O. H. *op. cit.*, 780–786.

⁶ Starch, D. "Transfer of Training in Arithmetical Operations," *Journal of Educational Psychology*, II, 306–310.

transfer of training in arithmetic. Eight subjects practiced for fourteen days on mental multiplication of three digit multiplicands by one digit multipliers. Before and after the practice they were given a series of tests as follows:

1. Eight problems in adding fractions.
2. Eight problems in adding two numbers of three digits each.
3. Test on memory span for numbers.
4. Eight problems in subtracting three-place numbers.
5. Eight problems in multiplying a four-place number by one digit.
6. Test on memory span for words.
7. Eight problems in multiplying a two-place number by one digit.
8. Eight problems of dividing a three-place number by one digit.

All calculations were performed mentally. A different set of tests was used before and after the practice. For comparison, seven other subjects were given the preliminary and final tests without the practice series. There was little change in the memory span for either group. The practiced subjects showed 20 to 40 per cent more improvement in the arithmetical tests than the unpracticed group. Starch concludes that "Training in one type of arithmetical operation improves very considerably the ability to do other fundamental arithmetical operations."

Poffenberger,¹ in 1915, reported an experiment on transfer in which he attempted to analyse the material used into stimulus and response. Eight subjects (in some cases eleven) repeated a series of seven tests five times. Four of these subjects were then selected as a training

¹ Poffenberger, A. T. "The Influence of Improvement in One Simple Mental Process upon Other Related Processes," *Journal of Educational Psychology*, VI (October, 1915), 459-474.

group while the others became the control group. The training group was then practiced for nine days on another set of four tests. On the tenth day, both groups were again tested on the original series of seven tests.

The seven tests in the test series were (1) Form Naming Test, (2) Adjective-Noun Test, (3) Cancellation of Groups containing 3 and 5, (4) Cancellation of groups containing 4 and 7, (5) Subtraction, (6) Multiplication, (7) Division. The four tests in the training series were (1) Color Naming Test, (2) Opposites Tests, (3) Cancellation of 3 and 5 separately, (4) Addition.

No significant transfer was found from the practice on the Color Naming Test to the Form Naming Test, from the Addition Test to the Subtraction or Division Tests, or from the Cancellation of 3's and 5's to the Cancellation of 4's and 7's. Positive transfer was found from the practice on Cancellation of 3 and 5 separately, to the Cancellation of the 3-5 group. Negative transfer was found from the practice on the Opposites Test to the Adjective-Noun Test, and from the Addition Test to the Multiplication Test.

Poffenberger concludes:

1. Where there are no identical bonds between stimulus and response in the two processes, the influence of one test upon the other will be neither positive nor negative . . .

2. Where there are identical elements in the two situations, or where a given process involves one or more bonds previously formed, there will be a positive or transfer effect . . .

3. Where one test necessitates the breaking of previously formed bonds and the formation of new ones, there will be a negative effect or an interference."¹

In 1915, Rugg investigated "The effect of a semesters training in descriptive geometry upon specific abilities in

¹ Poffenberger, A. T. op. cit., 474.

the mental manipulation of spatial elements, (a) of a strictly geometrical type; (b) of the quasi-geometrical type; (c) of a non-geometrical type."¹ The subjects, 413 college students, were divided into a training group (326 subjects), who took both the training course (descriptive geometry) and the two test-series; and a control group (87 subjects) who took only the two test-series. The two test-series were designed to measure efficiency in the mental manipulation of spatial elements. They cannot be described here for lack of space. Rugg's conclusions follow:

The study of descriptive geometry . . . operates:

1. Substantially to increase the students' ability in problems requiring the mental manipulation of spatial elements of a geometrical nature, the content of which are distinctly different from the visual content of descriptive geometry itself.

2. Substantially to increase the students' ability in solving problems requiring the mental manipulation of spatial elements of a slightly geometrical character . . .

3. Substantially to increase the students' ability in solving problems requiring the mental manipulation of spatial elements of a completely non-geometrical nature . . .

4. The training effect of such study in descriptive geometry operates more efficiently in those problems whose visual content more closely resembles that of the training course itself.²

Knight and Setzafandt,³ in 1924, reported a study, previously quoted, that showed transfer in the narrow mental function of the addition of fractions. They took two groups just ready to add fractions and tested them to make sure adding fractions was an accomplishment they did not possess. They then divided them into two

¹ Rugg, H. O. *The Experimental Determination of Mental Discipline in School Studies*, 25. Baltimore: Warwick and York, 1916.

² Rugg, H. O. *op. cit.*, 114.

³ Knight, F. B., and Setzafandt, A. O. H. *op. cit.*, 780-784.

groups (A and B) as nearly equal in ability as possible. Both groups were given the same amount of practice in the addition of fractions. Both were given the same type of instruction. The practice material for group A was so constructed that the numbers 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 21, 24, 28 and 30 were present in the denominators. The material for group B was similar with the exception that the numbers 3, 5, 7, 9, 14, 15, 18, 21, 28 and 30 did not appear at all as denominators. Group A had the same total amount of practice as Group B, but practiced any one denominator fewer times.

Following the period of instruction and drill, two tests were given to both groups on successive days. Each test was divided into two parts, the first part made up of problems involving denominators practiced by both groups, the second part of problems containing the denominators practiced by Group A only.

There was a loss in accomplishment on the second part of each test as compared to the first for both groups. The loss for the two groups was very nearly proportional, however, and was probably due to the inherent difficulty of the denominators in the second part. A comparison of the amount of accomplishment shows a high degree of transfer from the familiar to the unfamiliar materials for the group with limited training.

The authors account for the fact that the pupils worked with unfamiliar denominators about as well as with familiar denominators, as follows: "(1) The common denominator idea transfers from one group of denominator combinations to another with great ease . . . (2) The knowledge of relations between whole numbers transferred over to the use of these numbers in the denominator situation."¹

¹ Knight, F. B., and Setzafandt, A. O. H. *op. cit.*, 786-787.

In 1916 Rugg¹ made a careful survey of thirty transfer experiments and determined the relative amounts of transfer obtained. Orata² made a summary of the table given by Rugg and found that 23.25 per cent of the experiments included gave clear evidence of considerable transfer, and an additional 39.35 per cent showed indications of slight transfer. Orata³ made a similar summary of experiments from 1890 to 1927 and found that 32.32 per cent of these experiments showed considerable transfer and an additional 49.50 per cent showed appreciable transfer.

5. *How Transfer Occurs.*—That transfer occurs in some cases and not in others is amply proved by the experiments cited above. It remains, therefore, to consider under what conditions transfer occurs and how it takes place. One theory, and probably the one most commonly quoted, is Thorndike's theory of identical elements. This was stated in the 1903 edition of his *Educational Psychology* as follows:

The answer which I shall try to defend is that a change in one function alters any other only in so far as the two functions have as factors identical elements . . . By identical elements are meant mental processes which have the same cell action in the brain as their physical correlate.⁴

Ten years later he writes:

The general theory of identical elements—that one ability is improved by the exercise of another only when the neurones whose action the former represents are actually altered in the course

¹ Rugg, H. O. op. cit., 18.

² Orata, P. T. *The Theory of Identical Elements*, 40. Columbus: Ohio State University Press, 1928.

³ Orata, P. T. op. cit., 41.

⁴ Thorndike, E. L. *Educational Psychology*, 80-81. New York: Lemcke and Buechner, 1903.

of the exercise of the latter—is sound, and is useful in guiding thought.¹

Many educators today accept this theory of common elements, at least as a working hypothesis. Knight and Setzafandt state it in different terms when they write “The present status of the transfer problem seems to be: transfer exists to the extent that the same skills are used in the new situation.”² Poffenberger³ also used this theory of identical elements as the basis of his experiment.

Bagley suggests that what we carry over from one field to another is not a generalized habit or capacity but a generalized *ideal*. He writes:

But that there is some link that joins all specific habits of neatness is perfectly apparent to any one who may have a particularly “tidy” acquaintance.

What, then, is the connecting link between habits of different species and the same genus? The distinction already noted between habit and judgment suggests that, just as the latter may initiate the former, so judgment may connect and establish a functional relation between two specific habits. In other words, what I carry over from my school work to my farm work is not a generalized *habit* of work, but a generalized *ideal* of work . . . This ideal furnishes a motive and this motive holds me to conscious, persistent effort until the new habit has become effective.⁴

Rugg interpreted his finding, in the experiment previously described, as follows:

By one school of specialists, training has been regarded as specific in effect and transfer has been explained as due to “identical ability conditioning factors.” However, the typical attitude taken today is that practice may be generalized and transferred

¹ Thorndike, E. L. *Educational Psychology*, II, 417. New York: Teachers College, Columbia University, 1913.

² Knight, F. B., and Setzafandt, A. O. H. *op. cit.*, 781.

³ Poffenberger, A. T. *loc. cit.*

⁴ Bagley, W. C. *The Educative Process*, 212-213. New York: Macmillan, 1907.

through such factors as (1) ideational factors, (2) attention factors, (3) attitudinal factors. Thus, transfer is possible with central functions through the generalization of various ones of these factors. The emphasis here is on making the method of learning a conscious matter, the conscious organization of methods of procedure, the conscious utilization of methods of improvement, better understanding of how to use mental tools, rather than any transferable change (through practice) in the constitution of the organism itself.¹

Judd finds the theory of identical elements inadequate and suggests that transfer depends on the *generalization of experience*. He writes:

When one begins to define the identical elements of a situation in these broad general terms, he sees that the formula, identical elements of experience, is absolutely worthless . . . On the other hand, one is convinced as he considers carefully this general doctrine of identical elements that one of the most important considerations in any system of training is to help students to discover the possibilities of generalization presented in successive situations . . . The facts which are accumulated in a course in physics are capable of being carried over into the world of mechanics and the world of practical affairs. Let us take these facts, therefore, and see how far they can be generalized and made useful in all of the different spheres of experience. The problem of education thus turns out to be the problem of generalizing experience . . .

There is no inherent reason in the psychology of the individual mind or in the psychology of any subject of instruction for supposing that experience cannot be generalized. On the other hand, there is no reason to assume that experience of any type will infallibly carry over into any other sphere whatsoever. The generalization of experience is a qualitatively new fact wherever it appears . . . Everywhere in human experience there are large possibilities of generalizing experience, and everywhere in school there is danger that experience will be narrowly specialized.²

¹ Rugg, H. O. op. cit., 24.

² Judd, C. H. *Psychology of High School Subjects*, 419-420 Boston: Ginn and Company, 1915.

In the same way the student discovers a general principle of language structure . . . he is not merely taking an element that appears in one example . . . and in the second example . . . , and so on, recognizing an element common to all of these different situations; he is rather learning to extract from a variety of experiences a general principle or rule. The discovery of this general principle or rule is a new performance; it is an expression of the power of generalization. The cultivation of this power of generalization is the most important achievement in the student's education. It will not come without special endeavor on the part of the student and on the part of the teacher.¹

The different attempts to account for transfer are almost as numerous as the experiments dealing with the problem. Orata² gives a long and detailed analysis of the theories advanced by different experimenters.

6. *Methods of Facilitating Transfer*.—Closely related to the question of how transfer occurs is the even more important question, from the practical point of view, as to whether it is possible in any way to insure transfer and to increase the amount. The importance of this question is well stated by Woodrow when he writes:

When an experimental study has shown that practice in one operation, e.g., memorizing poetry, fails to produce improvement in another, e.g., memorizing prose,³ does this mean that the same conclusion is valid no matter what type of practice is used? . . . It is worth while to know that an experimenter allowed his subjects practice of a sort which did not result in a large degree of transference; but is it not of at least equal value to know how training can be given so as to transfer, that is, so as to produce improvement in a number of related functions?⁴

¹ Judd, C. H. op. cit., 432.

² Orata, P. T. op. cit., 47-51.

³ For such an experimental study, see Sleight, W. J., loc. cit. This experiment is summarized on page 3 of the present study.

⁴ Woodrow, H. "The Effect of Type of Training upon Transference," *Journal of Educational Psychology*, XVIII (March, 1927), 159.

Bagley, in 1907, reported an experiment by Carrie R. Squire, as follows:

At the Montana State Normal College careful experiments were undertaken to determine whether the habit of producing neat papers in arithmetic will function with reference to neat written work in other studies; the tests were confined to the intermediate grades. The results are almost startling in their failure to show the slightest improvement in language and spelling papers, although the improvement in the arithmetic papers was noticeable from the very first.¹

Ruediger,² however, in 1908, found that, while teaching the habit of neatness in one school subject, it was possible, by inculcating the ideal of neatness, to obtain an improvement in neatness in other school subjects. The experiment was conducted with seventh-grade pupils. Part of his instructions to the teachers were as follows:

1. In the written work of one school subject pay all the attention you can both to the habit and the ideal of neatness . . .
2. Talk frequently *with* the class (not *to*) on the importance of neatness in dress, business, the home, hospitals, etc., connecting it as far as you can with the subject under experiment . . .
3. Do *not* bring up the subject of neatness in connection with the other studies of the school . . .

It has also been suggested that the transfer of a certain method of procedure from one set of conditions to another, similar to but differing somewhat from the first, might be facilitated by rationalizing the process, i.e., by giving the subjects a theoretical explanation of principles underlying the method. Judd, in 1908, described an experiment by himself and Scholckow which

¹ Bagley, W. C. *op. cit.*, 208.

² Ruediger, W. C., "The Indirect Improvement of Mental Functions through Ideals," *Educational Review*, XXXVI, 364-371.

illustrates this point. The experiment was carried out with boys in grades 5 and 6. Judd reports as follows:

One group of boys was given a full theoretical explanation of refraction. The other group of boys was left to work out experience without theoretical training. These two groups began practice with the target under twelve inches of water. It is a very striking fact that in the first series of trials the boys who knew the theory of refraction and those who did not, gave about the same results . . . All the boys had to learn how to use the dart, and theory proved to be no substitute for practice. At this point the conditions were changed. The twelve inches of water were reduced to four. The differences between the two groups of boys now came out very strikingly. The boys without theory were very much confused. The practice gained with twelve inches of water did not help them with four inches. Their errors were large and persistent. On the other hand, the boys who had the theory, fitted themselves to four inches very rapidly.¹

Ruger,² in 1910, in his experiment on the solution of mechanical and geometrical puzzles, observed not only the results but also the method of transfer. It would take too much space to describe here the puzzles used and the method of procedure, so only a few of his conclusions will be quoted; and a few cases given to illustrate these conclusions. Four of his conclusions concerning the conditions for transfer of motor habits were as follows:

1. The mere presence, in the case of change of conditions, of motor habits appropriate to the new conditions did not necessitate positive transfer. It could co-exist with negative transfer.
2. The degree of positive transfer varied directly with the precision of analysis of the similarity of the new case to the old . . .
3. In some cases a generalized formula developed in connection with the first case was essential to effective transfer of motor habits to later modifications of the first case.

¹ Judd, C. H. "The Relation of Special Training to General Intelligence," *Educational Review*, XXXVI, 36-37.

² Ruger, H. A. "The Psychology of Efficiency," *Archives of Psychology*, No. 15.

4. Transfer was more effective in those cases where the formula or general rule was developed in the first few trials . . . than when the generalization had been arrived at after those habits had been set up.¹

The following cases, described by Ruger, will serve to illustrate these points:

1. A certain puzzle was so arranged that it could be presented in various forms. The manipulations for these various forms could all be comprised under a single formula. This general formula could be deduced from any one of these special forms. A number of subjects were tried with this puzzle. As soon as skill was acquired in dealing with one form of the puzzle it was changed to another form. The subjects who developed the general formula during the solution of the first form were able to use the specialized habits built up in the first form in the second. Those who formed merely the special habits without developing the principle attempted to carry over the habits without modification and were greatly embarrassed by the change.

2. A subject was tested with a puzzle in a given form. Then all the motor habits necessary for the rapid solution of this form were built up by practise on the separate acts of manipulation involved. The elements were organically related in the successive forms of the practise series, so that the practise was not on separate elements merely but on their connections. At the close of the practise series the subject was given the complete form, which was identical with that of the initial test. This form was not recognized as being related to the practice series, and the habits built up there were not brought into use.²

Within the last few years there have been a number of studies of methods of facilitating transfer. Haskell,³

¹ Ruger, H. A. op. cit., 87-88.

² Ruger, H. A. op. cit., 18-19.

³ Haskell, R. I. *A Statistical Study of the Comparative Results Produced by Teaching Derivation in the Ninth Grade English Classes of Non-Latin Pupils in Four Philadelphia High Schools*, Philadelphia: Doctor's thesis, 1923.

in 1923, and Hamblin,¹ in 1925, studied "the extent to which the effect of the study of Latin upon knowledge of English derivatives can be increased by conscious adaptation of content and method to the attainment of this objective." The procedure and conclusions of the two studies were essentially the same. Orata summarizes Hamblin's experiment as follows:

Two equivalent groups on the basis of I.Q. were established. The control groups or classes were taught Latin along the usual lines followed in most schools, little, if any, attention being paid to derivation of words. The experimental class or classes were taught in such a way that derivative work based upon a specific content and specific method was a conscious and definite aim of instruction . . . The results are overwhelmingly in favor of the experimental group . . . The author concludes that 'pupils devoting approximately one-fifth of the total class time to systematic work in derivation . . . may be expected to gain about three times as many Latin-derived words as Latin pupils not so taught.'²

Coxe³ attempted to prove the validity of an objective in Latin as suggested by the American Classical League, namely, an "increased ability to spell English words of Latin derivation." Five groups were used as follows, made up of classes in beginning Latin and of the same grade.

Group I (Latin Control Group). No effort was made to relate Latin to English spelling.

¹ Hamblin, A. O. *An Investigation to Determine the Extent to which the Effect of the Study of Latin upon Knowledge of English Derivatives Can be Increased by Conscious Adaptation of Content and Method to the Attainment of this Objective*. Philadelphia: Doctor's thesis, 1925.

² Orata, P. T. op. cit., 137.

³ Coxe, W. W. "The Influence of Latin on Spelling of English Words," *Journal of Educational Research Monograph*, No. 7.

Group II (Latin Experimental Group Alpha). Definite material was used to teach spelling. Teachers pointed out the similarity in spelling between the words in practice list and the Latin original place which the pupils were studying.

Group III (Latin Experimental Group Beta). Here the teacher was expected to lead the pupils to develop rules and principles, besides pointing out the similarity of spelling between the Latin word and its derivative.

Group IV (English, non-Latin, Control Group). No special effort was made to teach spelling.

Group V (Non-Latin Experimental Group). Used same material as prescribed for Groups II and III, but followed a method which did not presuppose any knowledge of Latin and followed best current practice in teaching spelling.

The author comes to the following conclusions:

1. Latin as ordinarily taught in our schools today does not improve the spelling of non-Latin words.

2. The spelling of words derived from Latin can be improved through the study of Latin . . . Merely pointing out the similarities in spelling between the Latin word and its derivatives, produces considerable gain, but not as much as when rules and principles are used.¹

In 1924, Johnson² studied the problem "Can pupils of geometry be taught to prove theorems more economically and effectively when trained to use consciously a technique of logical thinking; and furthermore, does such training, more than the usual method increase the pupils' ability to analyse and see relationships in other non-geometrical situations?" Two groups were used, the control group took geometry, but the experimental group was trained to use consciously a technique of logical thinking, whereas the control group was taught in the ordinary way. At the conclusion of the training,

¹ Coxe, W. W. op. cit., 107-108.

² Johnson, Elsie P. "Teaching Pupils the Conscious Use of a Technique of Thinking," *Mathematics Teacher*, XVII, 191-201.

each group was given an original exercise to test geometrical thinking, and the Monroe Silent Reading Test as a test of non-geometrical reasoning. The author concludes that:

These data would seem to offer conclusive evidence . . . that when pupils are taught to use, consciously, a technique of logical thinking, they try more varied methods of attack, reject more erroneous suggestions more readily, and without becoming discouraged maintain an attitude of suspended judgment until the method has been shown to be correct.

Furthermore, the results in tests of reasoning other than geometrical, according to the author:

would seem to indicate that such training in logical thinking with the materials of geometry tends to carry over these methods of attack and these attitudes to other problem situations not concerned with geometry.¹

Meredith, in 1927, reports an experiment carried out in 1925. According to the author, "Its aim was to test which of two different methods of teaching science was the more effective in securing transfer from training in defining scientific terms to defining ordinary terms in daily use, the two methods differing by the fact that one involved explicit reference to the form which a correct definition should take, while the other did not."²

Sixty boys (ages 13-14) were divided into three equal groups (A, B and C) of approximately equal average intelligence. The initial and final test series consisted of two definition tests. The training series given to groups B and C, consisted of three lessons on elementary magnetism. Group C was *trained* in definition as well as practiced, whereas Group B was only practiced.

¹ Johnson, E. P. *op. cit.*, 200-201.

² Meredith, G. P. "Consciousness of Method as a Means of Transfer of Training," *Forum of Education*, V (1927), 38.

Group B, on the final test showed no significant superiority to Group A, while Group C showed substantial gain. The author concludes "We are accordingly justified in affirming that Group C benefited in its training in defining terms and carried this training over or 'transferred' it to the definition of ordinary terms."¹

Woodrow,² in 1927, showed that, while subjects given undirected drill in memorizing certain kinds of material benefited little when they turned to new kinds of memorizing, other subjects, who were given training in a general technique of memorizing, benefited greatly.

Three groups of university students were employed in the experiment. The control group, consisting of 106 students, was given no practice or training but was tested in six forms of memorizing at the beginning and end of a period of four weeks and five days. The practice group, composed of 34 students, was given the same tests at the beginning and at the end and in the meantime was given routine practice in memorizing. The training group was given the same tests and practice as the practice group and in addition was given instruction in the technique of memorizing. According to the author:

The training group did much better than the practice group in every one of the six end tests . . . The percentage of gain in the end tests averaged 31.6 higher for the training group than for the control group . . . The experiment shows that in a case where one kind of training—undirected drill—produces amounts of transfer which are sometimes positive and sometimes negative, but always small, another kind of training with the same material may result in a transference, the effects of which are uniformly large and positive.³

¹ Meredith, G. P. *op. cit.*, p. 43.

² Woodrow, H. "The Effect of Type of Training upon Transference," *The Journal of Educational Psychology*, XVIII (March, 1927) 159-172.

³ Woodrow, H. *op. cit.*, 171.

These experiments suggest that, under certain conditions at least, transfer can be facilitated in the following ways:

1. By the formation of a generalized ideal.
2. By rationalizing the method of procedure, i.e., by a consideration of the underlying principles.
3. By generalizing the method of procedure, i.e., by the formulation of a general rule.
4. By pointing out identical elements in the new and the old situation and by giving training in recognizing such elements.

The above summary of the literature on transfer of training makes no pretence of being exhaustive. The purpose has been to refer to only enough investigations to make clear the present status of our knowledge concerning the question. For a more complete summary and extensive bibliographies the reader is referred to Orata.¹ Attention should also be called to the recent summary by Buswell and Judd² of experiments relating particularly to arithmetic.

II. NEED OF FURTHER EXPERIMENTATION

The above summary not only shows that much has already been done but it also indicates that further experimentation is needed, particularly to measure the amounts of transfer to be expected in the case of such functions as are specifically trained in the schools, and to determine methods of increasing this transfer.

1. *Unsolved Problems*.—Knight and Setzafandt analyze the problems that remain to be solved as follows:

¹ Orata, P. T. *op. cit.*

² Buswell, G. T. and Judd, C. H. *Summary of Educational Investigations Relating to Arithmetic*, 150-152. Chicago: University of Chicago Press, 1925.

The problem of transfer at the present time is concerned not with the omnipresence of transfer or its utter absence but with such questions as the following: (1) In what types of skill is there enough transfer to use? (2) On what level of mastery of a complicated skill does transfer in useful amounts begin to operate? (3) In what ways can transfer be facilitated or increased? (4) Are transferred skills or skills acquired in one set of data as strong, as lasting, and as easily operated in another set of data? How much is *lost* by transfer? (5) Is the amount of loss of skill from the original situation to the new situation a function of the dissimilarity of the new situation to the original situation?¹

Woodrow writes:

To determine the amount of transference that did result from the study of a particular subject is one thing, to determine the amount of transference which *might* be secured from the study is a different and far more difficult thing. The investigation of problems of the latter sort must constitute an important part of the program for establishing a scientific basis for methods of teaching.²

Meredith also refers to the need of further experimentation. He writes:

The widespread interest formerly shown in the problem of "Transfer of Training" has waned somewhat in recent years. Yet a great deal more evidence is surely necessary before the question can be regarded as closed, even in its theoretical aspects; and it cannot be regarded as solved in its practical (i.e., educational) aspect until the solution so far obtained has had its due influence on pedagogical practice . . . A great deal remains to be done to consolidate the position so far reached. The problem of formal discipline is one of the root problems of education, and many experiments under classroom conditions are necessary before the teacher can rightly interpret the psychologists solution of the problem of transfer.³

¹ Knight, F. B. and Setzafandt, A. O. H. op. cit., 780-781.

² Woodrow, H. op. cit., 172.

³ Meredith, G. P. op. cit., p. 37.

Finally, Orata, in 1928, after an exhaustive summary and analysis of transfer experiments up to that date, states:

In conclusion, the writer wishes to state that the theory of transfer which he has prepared is an interpretation that needs to be tested out more fully in actual practice by a thorough-going and adequately controlled experimentation and open-minded observation of actual results in the classroom . . . Emphasis should be placed upon the method of training the subjects for transfer for the purpose of discovering the most favorable conditions of transfer.¹

2. *Limitations of Previous Experiments.*—Although the experiments summarized above show that transfer takes place in some cases and does not in others, and give a clue as to the conditions under which to expect transfer and how it may be facilitated, they do not give definite answers to all of the practical problems confronting educators today, such as those outlined above by Knight and Setzafandt. There are several reasons for this.

1. Some of the experiments are inconclusive because of the small number of subjects used. Starch used only fifteen subjects and Poffenberger eight (in some cases eleven).

2. Many of the experiments were performed under laboratory conditions. As a result they may or may not give a true picture of what we may expect and what we may be able to accomplish under school conditions. The studies of Thorndike and Woodworth, Starch, Poffenberger, Judd, Ruger and Woodrow were all laboratory experiments.

3. Many of them dealt with situations that are foreign to our schools. Thorndike and Woodworth dealt with the estimation of areas, lengths, and weights, Judd with hitting a target under water, and Ruger with puzzles.

¹ Orata, P. T. op. cit., 178.

4. Those dealing with school situations are so few that they do not begin to cover the wide and complex field. Furthermore, most of the recent studies, particularly those dealing with the important question of how we can facilitate transfer, have been in subjects other than arithmetic, which is the subject with which this study is particularly concerned. Haskell, Hamblin and Coxe all dealt with Latin, Johnson with geometry, and Meredith with science.

5. Most of the experiments dealing with school situations attempted to measure the transfer between two complex and widely differing functions such as from accuracy in numerical computation to accuracy in arithmetical reasoning (Winch), from Latin to knowledge of English derivatives (Haskell and Hamblin), from Latin to ability to spell English words (Coxe), from reasoning in geometry to ability to reason in non-geometrical situations (Johnson), and from study of definitions in science to defining non-scientific terms (Meredith). While such experiments are exceedingly valuable, we also need information concerning transfer *between the various parts of a given subject*. There is a dearth of experiments, such as that of Knight and Setzafandt, dealing with transfer within a narrow field, although it is precisely in such cases that we would expect the largest and, as a result, the most useful transfer.

6. Most of the recent experiments dealing with methods of increasing transfer have had to do with high school and college students. Haskell, Hamblin, Coxe, Johnson and Meredith employed subjects of high school age, while Woodrow used college students. The results of these experiments, therefore, leave unanswered the important question: *How early in the grades can the meth-*

ods of teaching employed in these experiments be used to insure transfer and to increase the amount?

3. *Pedagogical Importance of the Problem.*—Closely related to the problem of transfer of training in arithmetic is the question whether there is such a thing as “ability in arithmetic.” As early as 1903 Thorndike and Fox¹ made a study of the arithmetical abilities of high school pupils. From this and other related researches, Thorndike concludes that “ability in arithmetic is but an abstract name for a number of partially independent abilities.”²

This question was further investigated by Stone, in 1908. In the case of 500 pupils in Grade 6A he found a correlation of .32 between arithmetical reasoning and fundamentals, and correlations between the fundamentals varying from .50, for addition with subtraction, to .95, for multiplication with division. He concludes that:

The single word that best describes the nature of the product of the first six years of arithmetical work is *complex* . . . The net result . . . is *several products* rather than *a product*. The study called arithmetic makes demands on a plurality of abilities. Hence it is inaccurate to speak of the arithmetical ability of pupils, and it is bad educational practice to treat the subject as though it were a unity instead of a plurality.³

A more recent investigator, Collar, comes to somewhat different conclusions. For pupils in Standard 5 the

¹ Thorndike, E. L., and Fox, W. A. “The Relationship Between The Different Abilities Involved in Arithmetic,” *Columbia University Contributions to Philosophy, Psychology and Education*, XI—2, 32–40.

² Thorndike, E. L. *Educational Psychology*, 32. New York: Lemcke and Buechner, 1903.

³ Stone, C. W. *Arithmetical Abilities and Some Factors Determining Them*, 40–43, New York: Teachers College, Columbia University, 1908.

author finds an average correlation of .73 between the fundamentals and of .48 between the fundamentals and problems. His most important conclusion is that "The evidence of the survey is distinctly in favour of the proposition that there is a single specific mental factor, operative in all kinds of mental work."¹

As a result of the overthrow of the belief in general transfer, and the doubt cast upon the existence of a single arithmetical ability, there has been a great growth in the complexity of our textbooks in arithmetic. For example, the older books taught, in each of the four processes, only the so-called "forty-five combinations," i.e., the forty-five possible combinations of the digits from 1 to 9; or a total of only 180 different facts. The newer textbooks teach many more than this number. Osburn² gives an excellent analysis of the facts involved in each of the four fundamental processes. He finds that altogether there are 1680 facts that the pupils may need to know and 1237 that they are almost sure to need after leaving school.

Osburn takes the position that each of these 1680 facts must be taught and practiced as individual facts and denies the adequacy of transfer to reduce the number. In discussing the addition facts used in multiplication, he writes: "No teacher can expect high accuracy in carrying in multiplication unless she has carefully and consistently taught each one of these 175 addition combinations."³ With respect to transfer he writes:

The real magnitude of this undertaking has been largely obscured by a pedagogical doctrine which we have thoughtlessly followed

¹ Collar, D. J. "A Statistical Survey of Arithmetical Ability," *British Journal of Psychology*, XI, 135-138.

² Osburn, W. J. *Corrective Arithmetic*, 10-25, 155-177. Boston: Houghton Mifflin, 1924.

³ Osburn, W. J. *op. cit.*, 18.

for ages. This doctrine is known as the "transfer of training." According to it, if a child is taught a combination in direct form, he will always know it in every form. For example, if a child is taught how much 8 and 2 are, it is assumed that he will always respond correctly to 2 and 8, 18 and 2, 28 and 2, 12 and 8, 22 and 8, and so on. In like manner a child who is taught that 8 from 13 equals 5 is expected to respond correctly to 8 from 23, 8 from 33, 5 from 13, 45 from 53, and so on. Lastly, a child who is taught how much 7 times 8 are is supposed to respond correctly to 8 times 7, 56 divided by 7, 62 divided by 8, and the like. Thus we have been encouraged to believe that all the work is done when we have taught 180 facts—the 45 so-called "principal combinations" in addition, subtraction, multiplication, and division. Just how this marvelous transfer is to take place completely from 180 combinations to the 1680 has never been explained. Yet teachers still go about their work with the simple faith that the expected will infallibly take place.¹

Osburn does not deny the possibility of transfer but apparently sees no hope of help in this direction. He writes:

There is indeed, a partial identity of elements in the additions 5 and 8 and 15 and 8. There is also partial identity of procedure and there may be identity of ideals . . . But the identity is never complete, and from this fact it necessarily follows that the transfer is also incomplete.²

Osburn realizes the magnitude of the task of teaching individually each of the 1680 facts and proposes to reduce it somewhat by eliminating all except those that the child is almost sure to need after he leaves school. As already noted, this, according to Osburn, would reduce the number to 1237. He recognizes that the teaching of even this number may be an impossible task when he writes: "If perchance any teacher becomes so busy that she can not possibly teach all of the 1237 facts,

¹ Osburn, W. J. op. cit., 11.

² Osburn, W. J. op. cit., 12.

she should teach the ones that are most difficult, hoping that the child, in some way or other, will learn the easy ones himself."¹

The old method of teaching 180 facts and trusting to marvelous and magical transfer to take care of the rest is hopeless, and probably no competent authority would advocate it today. On the other hand, the method of teaching as individual facts as many of the 1680 facts as time permits and "hoping that the child, in some way or other, will learn the easy ones for himself," does not appear much more promising, especially when we recall that the facts constitute but a small part of the fundamentals of arithmetic. The question naturally arises as to whether there may not be a middle course somewhere between these two extremes. Our present knowledge concerning the limitations of transfer certainly does not justify teaching a few facts and trusting blindly to transfer for the rest. On the other hand, what we know about the possibilities of transfer and methods of increasing the amount, certainly does not justify our rejecting all possibilities of assistance from this direction. Just because transfer has not taken place automatically in the past is no proof that it can not be secured by intelligent methods of teaching, and the fact that transfer is probably never complete is no justification for ignoring it, if it can be secured in useful amounts. At least the possibilities seem sufficiently promising to justify further experimentation along this line.

To make the questions raised in the preceding discussion somewhat more concrete, let us consider the teaching of the addition facts in greater detail. If the reverses and the combinations involving zero are included, there are 100 combinations of the digits from 0 to 9.

¹ Osburn, W. J. *op. cit.*, 21.

Besides these, in column addition and in carrying in multiplication, the pupils must know such sums as $14 + 3$, $24 + 3$, $18 + 5$, $28 + 5$ and so on. Osburn¹ lists 400 of these facts which are usually known as the higher decade combinations. Probably everyone would agree today that all of the 100 simple combinations must be memorized. This does not necessarily mean, however, that they must be taught and practiced as *separate* facts. A knowledge of $3 + 5$ does not insure a knowledge of $5 + 3$ but it may greatly facilitate the learning of this new fact; especially if the pupil has already, from previous experience with other similarly related facts, become acquainted with the general principle that the sum of two numbers is the same in whatever order they may be taken.

Since the higher decade combinations are closely related to the simple combinations, it may not be necessary to teach all of them as separate facts, providing a sufficient number are taught and in such a way as to make these relations clear to the children. After the pupil has learned $3 + 5 = 8$, $13 + 5 = 18$, $23 + 5 = 28$, $33 + 5 = 38$ and so on, can he not eventually generalize the process so that instruction and practice on further combinations in this series becomes unnecessary? Furthermore, would it not be possible in our teaching to aid the child in forming this generalization?

Knight indicates this possibility in the Twenty-ninth Yearbook of the Society for the Study of Education. In discussing the treatment of the higher decade combinations in eight current arithmetic texts, he writes:

The texts vary in their methodology of teaching—all the way from no instruction at all to specific identification of nearly one-half of the total number. Most texts are constructed on the theory

¹ Osburn, W. J. op. cit., 163-166.

that no attention need be paid to the teaching of higher decade combinations. In contrast to this practice of the majority a few texts supply direct instruction of varying amounts. Experimental data are wanting on the amount of instruction needed on these combinations. Probably all combinations need not be taught. The child can generalize and does indulge in inferential thinking.¹

The illustrations given above have been confined entirely to the teaching of the fundamental facts. Many others could be given to show that similar problems occur in connection with the teaching of the processes. Enough has been written, however, to show the need of further experimentation on transfer if we are to build up a scientific method in the teaching of arithmetic. We are in pressing need of authoritative and precise answers to questions such as the following: (1) How much transfer can we expect in a given case? (2) How can we, through our method of teaching, make sure of obtaining this transfer to a maximum degree? Until these questions are answered, and they can only be answered by a large number of experiments, we can only play safe by trying to teach all of the facts of a related series, and all of the minute details of a process. This not only involves a terrible waste of time and energy, but is such a gigantic task that it is doubtful if it can be successfully accomplished.

¹ Knight, F. B., *Twenty-Ninth Yearbook of the National Society for the Study of Education*, 193-194. Bloomington, Illinois. Public School Publishing Company, 1930.

CHAPTER II

PURPOSE AND METHOD

I. PURPOSE

The investigation here reported was planned to study the effect of instruction on three types of examples in two place addition upon the pupils' ability to handle closely related types in three place addition and in two and three place subtraction, and to determine whether the amount of transfer is a function of the method of teaching.

More specifically the problem was to determine whether the amount of such transfer can be increased by helping the pupils consciously to generalize the process and formulate, from the three types taught, a general method of procedure applicable to the related types; by rationalizing the process, that is, by considering the underlying principles; and by a combination of generalization and rationalization.

The following minor problems were also studied:

1. Is there a relationship between the amount of transfer and the mental ages of the pupils?
2. Are there sex differences in transfer?
3. Is the loss that occurs, in transfer from the specific type of example taught to the related types, a function of the dissimilarity between the types?

II. BRIEF DESCRIPTION OF METHOD AND MATERIALS

The following description of the methods and materials used in this experiment is intended to be as concise as is

consistent with clearness. More detailed information is given in the latter part of this chapter and in the Appendix, for those who are interested.

1. *Time and Place.*—In order to test the general plan and the materials used, the experiment was first tried out, early in the second semester 1927-28, in the second grade of the Training School of the Bowling Green State College, Bowling Green, Ohio. Later in the same year it was carried out with twenty second grades in the public schools of Toledo, Ohio. It was found that many of the pupils in Toledo were too far advanced for the purposes of the experiment so it was repeated late in the first semester of the following year (1928-29). At this time sixteen second grades in Toledo, one in Bowling Green and fourteen in Findlay, Ohio were used.

2. *Groups.*—In order to make it possible to study the effect upon transfer of generalization, rationalization, and generalization and rationalization combined, the grades were divided into four groups, each group being taught by a different method. At the conclusion of the experiment the pupils were separated into groups of four, each taught by a different method, and matched for sex, mental age, teachers' estimate of general ability and score made on the preliminary testing. With the number of records available, it was impossible to match the individual pupils for chronological age. The four groups as a whole, however, are rather closely balanced with respect to this trait. Full data concerning the matching of the pupils are given later in this chapter.

Altogether 112 of these matched "quartets" were formed, 53 composed of girls and 59 of boys. This resulted in four matched groups of 112 pupils each, and each taught by a different method of instruction. These constitute the four groups reported upon in this study.

They will be referred to as Groups A, B, C and D, and the corresponding methods of instruction as Methods A, B, C and D.

3. *The Test-series*.—All four groups were given the same *test-series*, consisting of three tests.¹ These were given four times, at the beginning, at the end and twice during the experiment. They contained fifty-six examples of twenty-nine different types, as follows:

Type 1.—Ten addition combinations.

4	5	2	6	5	7	4	7	8	6
2	3	2	1	2	1	3	2	1	2
—	—	—	—	—	—	—	—	—	—

Type 2.—One place addition, three addends.

2	4
2	2
3	1
—	—

Type 3.—One place addition, four addends.

2	4
2	2
3	2
1	1
—	—

Type 4.—Five subtraction combinations.

6	8	4	7	7
2	3	2	1	2
—	—	—	—	—

Type 5.—Two place addition, two addends.

45	26
23	22
—	—

¹ The tests are given in the Appendix.

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Type 6.—Same as Type 5, except the numbers were dictated to the pupils.

$$\begin{array}{r} 74 \\ 22 \\ \hline \end{array}$$

Type 7.—Two place addition, three addends.

$$\begin{array}{r} 52 \\ 32 \\ 13 \\ \hline \end{array} \qquad \begin{array}{r} 54 \\ 22 \\ 21 \\ \hline \end{array}$$

Type 8.—Same as Type 7, except the numbers were dictated.

$$\begin{array}{r} 22 \\ 22 \\ 23 \\ \hline \end{array}$$

Type 9.—Addition of a two place, a two place and a one place number in the order stated.

$$\begin{array}{r} 24 \\ 23 \\ 2 \\ \hline \end{array}$$

Type 10.—Two place addition, four addends.

$$\begin{array}{r} 24 \\ 22 \\ 22 \\ 11 \\ \hline \end{array} \qquad \begin{array}{r} 24 \\ 23 \\ 31 \\ 11 \\ \hline \end{array}$$

Type 11.—Three place addition, two addends.

$$\begin{array}{r} 274 \\ 213 \\ \hline \end{array} \qquad \begin{array}{r} 586 \\ 212 \\ \hline \end{array}$$

Type 12.—Same as Type 11, except the numbers were dictated.

$$\begin{array}{r} 542 \\ 232 \\ \hline \end{array}$$

Type 13.—Three place addition, three addends.

526	464
221	322
132	211
—	—

Type 14.—Three place addition, four addends.

242	242
222	232
322	213
111	211
—	—

Type 15.—Two place subtraction.

48	67
23	22
—	—

Type 16.—Same as Type 15, except the numbers were dictated.

68
23
—

Type 17.—Three place subtraction.

477	687
212	231
—	—

Type 18.—The addition of a two place, a one place and a two place number in the order stated.

52
2
22
—

Type 19.—The addition of a one place, a two place and a two place number in the order stated.

6
32
21
—

Type 20.—The addition of a one place, a two place and a one place number in the order stated.

$$\begin{array}{r} 4 \\ 32 \\ 1 \\ \hline \end{array}$$

Type 21.—The addition of a three place and a two place number.

$$\begin{array}{r} 357 \\ 21 \\ \hline \end{array}$$

Type 22.—The addition of a two place and a three place number.

$$\begin{array}{r} 24 \\ 322 \\ \hline \end{array}$$

Type 23.—The addition of a three place and a one place number.

$$\begin{array}{r} 235 \\ 3 \\ \hline \end{array}$$

Type 24.—The addition of a one place and a three place number.

$$\begin{array}{r} 6 \\ 342 \\ \hline \end{array}$$

Type 25.—The addition of a two place, a three place and a one place number in the order stated.

$$\begin{array}{r} 54 \\ 322 \\ 2 \\ \hline \end{array}$$

Type 26.—The subtraction of a one place from a two place number.

$$\begin{array}{r} 47 \\ 2 \\ \hline \end{array} \qquad \begin{array}{r} 58 \\ 3 \\ \hline \end{array}$$

Type 27.—The subtraction of a two place from a three place number.

$$\begin{array}{r} 348 \\ 23 \\ \hline \end{array} \qquad \begin{array}{r} 574 \\ 12 \\ \hline \end{array}$$

Type 28.—The subtraction of a one place from a three place number.

$$\begin{array}{r} 356 \\ 2 \\ \hline \end{array} \qquad \begin{array}{r} 148 \\ 3 \\ \hline \end{array}$$

Type 29.—Carrying in addition.¹

$$\begin{array}{r} 28 \\ 14 \\ \hline \end{array} \qquad \begin{array}{r} 49 \\ 25 \\ \hline \end{array}$$

4. *The Training.*—The training consisted of instruction and practice on three specific types of examples (1) the addition of two numbers of two digits each (Types 5 and 6), (2) the addition of three numbers of two digits each (Types 7 and 8), and (3) the addition of a two place, a two place and a one place number in the order stated (Type 9).

A different method of instruction was employed with each of the groups. The methods were as follows:

METHOD A.—The pupils were shown how to perform the process. There was no generalization or consideration of underlying principles. For example, in teaching the pupils how to add a two place, a two place and a one place number, in the order stated (Type 9), the teacher simply showed the pupils how to write and add the numbers.

$$\begin{array}{r} 52 \\ 32 \\ 3 \\ \hline \end{array}$$

¹ This type was not included in the tests given part of the pupils.

METHOD B.—The pupils were helped to formulate general methods of procedure from the specific types taught. These generalizations were constantly emphasized throughout the teaching. In teaching the type of example given above the teacher not only showed the pupils how to write the numbers, but also helped them to form the generalization that they must always be written so as to keep the right hand column straight. This will be called the *method of generalization*.

METHOD C.—The reasons and principles underlying the specific types taught were discussed with the pupils. The formulation of general rules of procedure was avoided as much as possible. In teaching the type of example given above the pupils discussed the principle that ones can only be added to ones and tens to tens, but nothing was said about keeping the right hand column straight. We will call this the method of *rationalization*.

METHOD D.—General methods of procedure were formulated and the underlying principles were discussed. In the type of example previously used as an illustration, the pupils were helped to the conclusion that the right hand column must be kept straight in order to add ones to ones and tens to tens. We will refer to this as the method of *generalization and rationalization*.

All four groups were given the same type of practice. This practice was partly abstract drill and partly practice in applying the abstract processes to concrete situations. The practice was confined to the specific types of examples taught (Types 5 to 9). The groups all received the same *total* amount of instruction and practice, but the methods of instruction employed with Groups B, C and D required more time than that used

with Group A. This resulted in less practice and more instruction for Groups B, C and D.

5. *Control Group*.—In most experiments of this type provision is made for a group which is given the test-series but is not given the training or practice. Such a group is usually called a control group. No group of this type is provided for in this experiment. There were two reasons for this. In the first place it was found to be impossible to secure such a group. To make the testing analogous to that of the other groups, it would have been necessary to give the test series four times, extending over a period of fifteen days. No second grade teacher was found who was willing to take this much of the pupils' time, especially since no instruction in arithmetic could have been given during the period.

In the second place no such group is essential to the primary purpose of this experiment, namely to measure the increase in transfer due to generalization, rationalization, and generalization and rationalization combined. For this purpose it is sufficient to compare each of Groups B, C and D to Group A, with which neither generalization nor rationalization was employed. Group A, therefore, becomes the *control group* for the purposes of this experiment.

6. *Pupils' Preliminary Knowledge*.—The experiment was planned to start when the pupils knew ten addition combinations (Type 1), one place addition with three addends (Type 2), one place addition with four addends (Type 3) and five subtraction combinations (Type 4); and before they knew anything about the addition and subtraction of two and three place numbers. As is usually the case with experiments carried out under class-room conditions, it was possible to meet these conditions only approximately. On the first testing the

448 pupils, whose records are reported in this experiment, worked correctly 87.7 per cent of the examples they had been taught before the experiment started (Types 1-4), 39.5 per cent of those they were later taught during the course of the experiment (Types 5-9); and 18.4 per cent of those not taught (Types 10-29).

7. *Time Schedule*.—The experiment took fifteen days, twenty minutes a day, eight days being used for testing and as much practice as time permitted, and seven days for instruction and practice. The complete time schedule was as follows:

First and Second Days.—First testing with test-series.

Third Day.—Instruction on the addition of two numbers of two digits each (Types 5 and 6).

Fourth and Fifth Days.—Practice on Types 5 and 6.

Sixth and Seventh Days.—Second testing.

Eighth Day.—Instruction on addition of three numbers of two digits each (Types 7 and 8).

Ninth Day.—Practice on Types 5, 6, 7 and 8.

Tenth and Eleventh Days.—Third testing.

Twelfth Day.—Instruction on addition of a two place, a two place and a one place number in the order stated (Type 9).

Thirteenth Day.—Practice on Types 5, 6, 7, 8 and 9.

Fourteenth and Fifteenth Days.—Fourth testing.

III. MATCHING THE GROUPS

The validity of the major results reported in this study, and of the conclusions drawn from these results, depends on the substantial equality of the four groups whose records are compared. In order to insure such substantial equality, the method of matched quartets was employed. The matching was exact only in the case of sex, so an intelligent interpretation of the results obtained makes it necessary to report in detail the degree of exactitude with which it was possible to match each of the

other traits. Five traits were considered in matching, namely sex, mental age, chronological age, score on preliminary testing, and teacher's estimate of general scholastic ability.

1. *Mental Age*.—Soon after the close of the experiment all pupils were given the Detroit Primary Intelligence Test, for Grades II, III and IV, Form D.¹ In Toledo these tests were all given by the same individual, in Findlay by a second individual and in Bowling Green by a third. All three were thoroughly familiar with intelligence testing and had had considerable experience in administering such tests.

The difference in mental ages between the different pupils in a "quartet" in no case exceeds three months. The complete distribution of these differences is given in Table I. This table shows that only 26 of the 336 differences, or 7.7 per cent, exceeded two months.

TABLE I.—DISTRIBUTION OF DIFFERENCES IN MENTAL AGES BETWEEN MEMBERS OF THE SAME QUARTET

Difference (months)	Frequency of difference		
	Between Methods A and B	Between Methods A and C	Between Methods A and D
3	12	6	8
2	22	30	28
1	39	48	45
0	39	28	31

The complete distribution of the mental ages of the 448 pupils is given in Table XLV, in the Appendix.

¹Baker, H. J. *Detroit Primary Intelligence Test*. Bloomington, Illinois: Public School Publishing Co.

This table is summarized in Table II which gives the mean, standard deviation and range for each group. An examination of this table shows that the greatest difference in mean mental ages between any two groups is .3 months, and the greatest difference in the standard deviations is .2.

TABLE II.—SUMMARY OF MENTAL AGES OF THE 448 PUPILS
(In Months)

	Group			
	A	B	C	D
Mean.....	100.5	100.6	100.8	100.8
S. D.....	10.2	10.3	10.2	10.1
Range.....	52	56	54	54

2. *Teachers' Estimates.*—Teachers' estimates of the pupils' general ability were used as a check on the mental ages obtained by the Detroit tests. It was originally planned to administer two intelligence tests, but it was found to be impossible to do this, as the school officials thought it would take too much of the pupils' time. The teachers' estimates were therefore substituted for the second intelligence test. In most cases these estimates are the average of all the grades given the pupil by the teacher at the end of the semester. In cases where such grades were not given, the teacher was asked to give a *general estimate of the pupils' work in all subjects*.

These teachers' estimates are here given in terms of the letters A, B, C, D and F, as this was the system employed in all of the schools participating in the experi-

ment. In matching the pupils, preference was given to the mental age as probably the more reliable measure of general ability. The difference in teachers' estimates of two pupils in the same quartet in no case exceeds two steps, as A to C, or B to D. The complete distribution of these differences is given in Table III. This table shows

TABLE III.—DISTRIBUTION OF DIFFERENCES IN TEACHERS' ESTIMATES BETWEEN MEMBERS OF THE SAME QUARTET

Difference (steps)	Frequency of difference		
	Between Methods A and B	Between Methods A and C	Between Methods A and D
2	2	5	2
1	66	57	60
0	44	50	50

that only nine of the 336 differences, or 2.7 per cent, exceeded one step, as A to B, or B to C.

Table XLVI, in the Appendix, gives the complete distribution of the teachers' estimates of the 448 pupils. The essential facts of this table are summarized in Table IV, which gives the mean, standard deviation and range for each group.

Since, in obtaining the means, B was assigned a value of 3 and C a value of 2, the means of all of the groups represent grades between B and C. If we use the marks D+, C+, etc., anything from 1.8 to 2.2 would represent a grade of C, anything from 2.3 to 2.7 a grade of C+ and from 2.8 to 3.2 a grade of B. The means of all of the groups would then represent a grade of C+.

3. *Initial Scores.*—In forming the quartets, if two or more pupils taught by a given method were nearly equal with respect to mental age and teacher's estimate preference was given to the pupil whose score on the first testing was most nearly equal to those of the other pupils in the quartet being formed. This resulted in a rough matching for score on the initial testing, although, with the number of papers available, it was not possible to

TABLE IV.—SUMMARY OF TEACHERS' ESTIMATES OF THE 448 PUPILS

	Group			
	A	B	C	D
Mean.....	2.7 ^a	2.4	2.4	2.5
S. D.....	9.5	8.2	9.0	8.0
Range.....	4 ^b	3 ^c	4	4

^a These means were found by assigning values of 4, 3, 2, 1 and 0 to the letters A, B, C, D and F, respectively, and averaging in the usual way.

^b 4 steps, i.e., from A to F.

^c 3 steps, i.e., from A to D.

make this matching as close as in the cases of mental age and teachers' estimate. Table V gives the distribution of the differences in initial scores between pupils in the same quartet.

The largest possible score on the initial testing was 56. Table V shows that 293, or 87.2 per cent of the differences were 10 or below, whereas 43, or 12.8 per cent were above 10.

Table XLVII, in the Appendix, gives the complete distribution of the initial scores of the 448 pupils. The essential facts of this table are summarized in Table VI

which gives the mean, standard deviation and range for each group. Although the individual pupils are only roughly matched for initial scores, an examination of

TABLE V.—DISTRIBUTION OF DIFFERENCES IN INITIAL SCORES BETWEEN MEMBERS OF THE SAME QUARTET

Difference (examples)	Frequency of difference		
	Between Methods A and B	Between Methods A and C	Between Methods A and D
26-30	0	0	1
21-25	2	0	0
16-20	3	5	4
11-15	8	12	8
6-10	26	30	29
0-5	73	65	70

Table VI shows that the four groups are rather closely balanced with respect to this trait.

TABLE VI.—SUMMARY OF INITIAL SCORES (IN EXAMPLES) OF THE 448 PUPILS

	Group			
	A	B	C	D
Mean.....	25.7	27.9	26.3	26.2
S. D.....	9.5	8.7	9.5	9.3
Range.....	38	38	39	36

4. *Chronological Ages*.—It was found to be impossible, with the number of records available, to match the

individual pupils for chronological age in addition to the other four traits. The attempt was made, however, to balance the four groups for this trait. How well this was done is shown in Table XLVIII, in the Appendix, which gives the complete distributions of the chronological ages for the different groups, and in Table VII which summarizes these distributions.

TABLE VII.—SUMMARY OF CHRONOLOGICAL AGES OF THE 448 PUPILS
(In Months)

	Group			
	A	B	C	D
Mean.....	96.2	97.2	96.7	95.7
S. D.....	6.6	6.5	6.9	6.8
Range.....	29	28	28	29

Table VII shows that the four groups were rather closely balanced with respect to chronological age, although it was not possible to match the individual pupils for this trait.

IV. TEST-SERIES

We have already described the various types of examples in the test-series, and the tests themselves are given in the Appendix. Three questions remain to be considered (1) the method of construction (2) the method of administration, and (3) the reliability of the tests.

1. *Method of Construction.*—The tests were so constructed that the only addition and subtraction facts

needed are the ten addition facts given at the beginning of Test No. 1 and the five subtraction facts given in Test No. 2. To illustrate, let us take the fourteenth example in Test No. 1, which is

4

2

2

1

—

Starting at the top, the pupil first needs to know 4,

2

—

next 6 and finally 8. All of these combinations are

2

—

1

—

included among the ten given in Test No. 1. In all of the instruction and practice during the experiment the pupils started adding at the top and this was also the method that had been previously taught to all of the pupils.

This method of constructing the tests was employed to obviate, in so far as possible, the question whether a pupil's failure to work a given example correctly was due to lack of knowledge of the method of procedure or lack of the necessary facts. The 448 pupils were 95.0 per cent accurate on the ten addition combinations, on the first testing and 98.0 per cent accurate on the final testing. They were 86.1 per cent accurate on the subtraction combinations on the first testing and 94.2 per cent accurate on the final testing. The comparatively low accuracy on the subtraction combinations on the first testing was largely due to the fact that a number of the pupils added instead of subtracting.

2. *Method of Administration.*—The tests were not administered as speed tests. The directions given to the teachers were: "Give the pupils all the time they can profitably use. When you are convinced that they have done all that they are able to do, take up the papers." The reason for this was that the purpose of the tests was to determine what types of examples the pupils were able to work, not to determine their speed.

In order to insure, in so far as possible, that the tests would be administered uniformly and in the way desired, the teachers were given detailed *Directions for Administering the Tests*.¹ These directions were also discussed with the teachers in the preliminary conference.

The purpose of Test No. 3 was to determine the pupils' ability properly to place numbers having different numbers of digits, in both addition and subtraction. The examples were, therefore, dictated by the teacher. In order to simplify the task of correcting the papers the pupils were provided with blank forms on which to work the examples.²

3. *Reliability.*—In order to determine the reliability of the tests, they were given to a group of fifty-three second grade pupils (twenty-six girls and twenty-seven boys) and repeated again after an interval of one day, on which no arithmetic was taught. The mean number of examples worked correctly on the first testing was 14.9, while on the second testing it was 15.3, a gain of only .4 of an example. The self-correlation between the two testings was .85, with a probable error of .03. These pupils were also given the Detroit Primary Intelligence Test, Form D, and the mean of their mental ages was found to be 90.5

¹ A copy of these directions will be found in the Appendix.

² A copy of this form is given in the Appendix.

months. The complete records of these fifty-three pupils are given in Table XLIX, in the Appendix.

V. THE TRAINING

The general character of the training given the different groups has already been described. Since the major purpose of the experiment was to compare the amounts of transfer obtained by the different methods, and since the actual teaching had to be in the hands of the regular class-room teachers, great pains were taken to insure the types of teaching desired. Two means were employed to accomplish this, (1) detailed plans for teaching were given the teachers and (2) at least one conference was held with all the teachers concerned.

1. *Directions for Teaching.*—Great care was taken in the preparation of the lesson plans given the teachers in order to insure clearness and teachability. After being prepared in their original form the plans were submitted to two experienced primary teachers for criticism and were then revised. They were next taught to small groups of pupils by the second grade critic in the Bowling Green State College. The writer observed much of this teaching. As a result of this trial a few further minor changes were made and the time necessary to teach the lessons was determined. A full set of the directions given to the teachers of the different groups is given in the Appendix.

2. *Conferences with Teachers.*—Before starting the experiment in Toledo a conference was held with all of the teachers concerned. The purpose of this conference was two-fold. In the first place considerable time was spent in discussing the purpose and general plan of the experiment as it was the belief of the author that, without a clear understanding of these, the teachers could not

intelligently carry out the plans furnished them. Next the plans themselves were discussed. Throughout the conference opportunities were given for questions and suggestions and many were asked and given. The teachers all showed an intelligent interest and a willingness to coöperate.

The author also had the assistance of a member of the Department of Tests and Measurements of the Toledo Schools, who was present at this conference and who supervised the carrying out of the experiment in Toledo.

Two conferences, similar to that described above, were held with the Findlay teachers, who showed an equal interest and willingness to coöperate. The work in Bowling Green was under the direct charge of the critic teacher previously mentioned.

VI. THE TEACHERS

The 448 pupils whose records are used in this experiment were taught by fifty different teachers. In selecting the teachers in Toledo an effort was made, in consultation with the supervisors, to secure teachers of equal ability for the four different groups. In Findlay all of the second grade teachers in the system were used. Four different teachers were used in Bowling Green. One was the critic teacher previously mentioned and the other three were student teachers in the State College.

Because of the large number of teachers, and of the attempt to equate the Toledo teachers, it is believed that there were no significant differences in the quality of the instruction given the four different groups, except such as were inherent in the different methods employed.

The experience of the teachers varied all the way from over twenty years to no experience except a few

weeks of practice teaching. The amount of professional training varied from none to a college degree, the mode being graduation from a two-year teacher training course. In short, the teachers were quite representative of the rank and file of teachers in our schools today and the results obtained, therefore, show what can be accomplished by such teachers.

VII. IMPORTANT FEATURES OF THIS EXPERIMENT

Before closing this discussion of purpose, methods, and materials, it seems advisable to summarize briefly the outstanding features of this experiment, as they appear to the author.

1. The experiment attempts to measure transfer within a narrow field—from certain types of examples in two place addition, to closely related types in two and three place addition and subtraction.

2. The problem is a school problem and, as we have already attempted to show, one of considerable importance in building up a scientific method in arithmetic. Our methods of teaching the fundamentals of arithmetic must be based upon (*a*) a denial of all transfer, (*b*) an assumption of complete transfer, or (*c*) an experimental knowledge of how much transfer may be expected in a given case and how it may be facilitated.

3. The experiment was carried out under ordinary school conditions by the regular class room teachers. The results obtained, therefore, show what can be done in the average school and the conclusions drawn from these results apply to such a school.

4. The experiment does not measure how much transfer was obtained by haphazard and more or less unknown methods of teaching, but shows how much can

be obtained by controlled methods of instruction definitely adapted to that end.

5. It is in the second and third grades that the multitudinous facts and the many different steps in the processes of arithmetic must be taught. It is, therefore, in these grades that a definite knowledge of what we can expect from transfer, and of how we can obtain it to the maximum degree, is most important. It was for this reason that the present experiment was planned for the second grade.

CHAPTER III

TABULATION AND DISCUSSION OF RESULTS

I. TRANSFER OF TRAINING

Before presenting results it is necessary to define what is meant by transfer of training. The term is used throughout this study in the sense defined by Ruger.

It has seemed advisable to the writer to use the term transfer in a very broad sense to include the effect of any given experience on any subsequent one whether the effect results directly or by means of an idea, whether the transfer is one of method, or of material, or of motor processes, and whether it is positive or negative.¹

1. *Evidence of Transfer.*—In order to see if the training that was given to the pupils on examples of Types 5 to 9 had any effect on their ability to work the remaining types (10 to 29), Table VIII was compiled, showing the percentage of examples of each type worked correctly on the first and last testings. A short notation is used in Table VIII to describe the various types of examples. To illustrate, the notation $1 + 1$ is used to designate the addition of two numbers of one digit each, $2 + 2 + 2$ to designate the addition of three numbers of two digits each, $3 - 3$ to represent the subtraction of a three place number from a three place number, etc. This short notation will be used throughout the remainder of this study to avoid the repetition of lengthy descriptions of the different types of examples.

¹ Ruger, H. A. op. cit., 85.

Table VIII shows that there was a decided improvement on Types 10 to 28, in spite of the fact that the pupils were given no instruction or drill on these types during the experiment. Type 29, the only type on which there was no substantial improvement, will be discussed later.

Obviously, an absence of improvement would have proved definitely that there was no transfer from the specific types taught. The fact that large improvement did occur, however, is not necessarily a proof of transfer, as it might be due to any or all of several causes. Part might be the result of accidental mistakes on the first testing that were corrected on the last. This would not be sufficient to account for any great improvement, however, as such accidental mistakes were as apt to occur on the last testing as on the first. Part might be due to the pupils having forgotten, on the first testing, some of the combinations involved. As these combinations were used throughout the experiment, errors due to this cause would naturally be less frequent on the last testing than on the first. It is evident that this could not account for the amount of improvement shown, however, as the pupils were 95.0 per cent accurate on the ten addition combinations used in the experiment, on the first testing, and 98.0 per cent accurate on the last testing; and they were 86.1 per cent accurate on the five subtraction combinations used in the experiment, on the first testing, and 94.2 per cent accurate on the last testing. The improvement on the subtraction combinations was largely due to the fact that a number of the pupils, on the first testing, added instead of subtracting, this error being corrected before the last testing. It is also possible that part of the improvement might be due to instruction outside of school. Unfortunately it was not possible

TABLE VIII.—PERCENTAGE OF EXAMPLES CORRECT ON FIRST AND LAST TESTINGS

Type of example		First testing	Last testing	
No.	Description			
1	$1 + 1^a$	95.0	98.0	Taught before beginning of experiment
2	$1 + 1 + 1^b$	87.2	96.8	
3	$1 + 1 + 1 + 1$	82.5	93.7	
4	$1 - 1$	86.1	94.2	
5	$2 + 2^c$	43.2	96.5	Taught during experiment
6	$2 + 2$ (Dictated)	51.1	96.4	
7	$2 + 2 + 2$	37.4	93.1	
8	$2 + 2 + 2$ (Dictated)	45.8	95.3	
9	$2 + 2 + 1^d$	19.9	93.3	
10	$2 + 2 + 2 + 2$	35.3	87.7	Not taught
11	$3 + 3$	36.3	93.5	
12	$3 + 3$ (Dictated)	35.5	85.7	
13	$3 + 3 + 3$	30.0	88.4	
14	$3 + 3 + 3 + 3$	27.1	82.6	
15	$2 - 2$	43.7	87.3	
16	$2 - 2$ (Dictated)	38.2	84.8	
17	$3 - 3$	35.7	84.4	
18	$2 + 1 + 2$	17.0	92.9	
19	$1 + 2 + 2$	13.6	76.8	
20	$1 + 2 + 1$	10.0	71.4	
21	$3 + 2$	3.3	56.2	
22	$2 + 3$	4.2	47.1	
23	$3 + 1$	7.8	64.7	
24	$1 + 3$	3.6	55.8	
25	$2 + 3 + 1$	2.5	46.0	
26	$2 - 1$	14.4	63.4	
27	$3 - 2$	2.1	47.3	
28	$3 - 1$	6.8	57.3	
29	Carrying in addition	1.3	4.1	

^a This notation represents the addition of two numbers of one digit each.

^b This notation represents the addition of three number of one digit each.

^c This notation represents the addition of two numbers of two digits each.

^d This notation represents the addition of two numbers of two digits each and a number of one digit, the latter coming last (at the bottom).

to get any data as to the amount of this, if any. Finally the improvement might be the result of a spread, or transfer, from the instruction and practice given on related types of examples.

In an effort to determine whether the improvement was due to outside instruction, or to transfer, the two examples of Type 29 (carrying in addition) were added to Test No. 1, after the experiment had been carried out with several groups of pupils. These examples appeared on the tests taken by 317 of the 448 pupils. If the parents, or any one except the teacher, gave any instruction at all on addition there would be no reason to suppose that they would not, in many cases, include carrying. In such a case it would be reasonable to expect an appreciable improvement on this type of work. If, on the other hand, the improvement was due to transfer, no such improvement could be reasonably expected, as nothing was taught that would help the pupils in the solution of this type of example. Table VIII shows that the 317 pupils, who took the tests involving carrying, solved 1.3 per cent of such examples on the first testing, and 4.1 per cent on the last. An examination of the records of the 317 pupils showed that five pupils worked eight examples on the first testing and sixteen pupils worked twenty-six examples on the last testing. This does not indicate much outside instruction, at least on carrying in addition. Further evidence will be presented later to show that the improvement was probably largely due to transfer.

2. *Method Employed to Measure Transfer.*—The measure of transfer used in this study is the *percentage of maximum possible transfer*, or, briefly, the *percentage of transfer*. This was obtained by dividing the number of examples to which the effects of the specific training

spread, by the total number to which it might have spread, had the transfer been complete. This is illustrated graphically in Figure 1, which shows the three groups of examples as they would be just before the second testing. At this time, twenty-two of the exam-

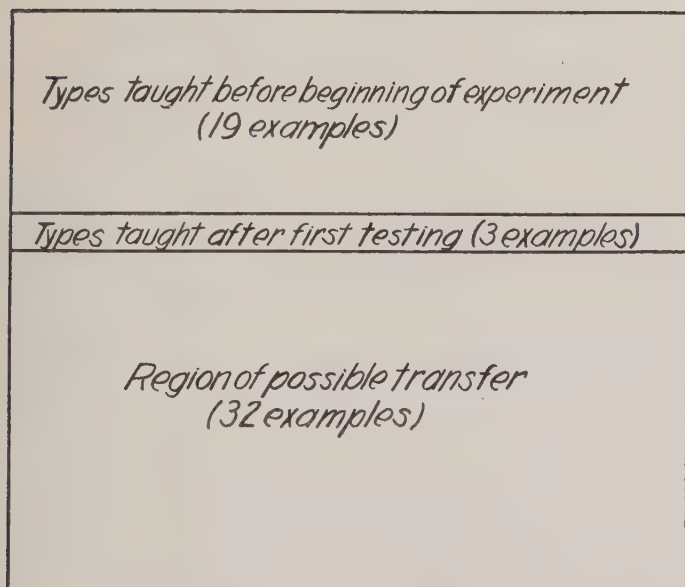


FIG. 1.—The three types of examples just before the second testing.

ples belong to types that have been taught, nineteen before the beginning of the experiment and three after the first testing. The remaining thirty-two examples are of types that have never been taught and, therefore, represent the field of possible transfer, or the examples to which the specific training just received by the pupils may spread.

To illustrate, suppose a pupil, on the first testing, was unable to work any of the thirty-two examples in the field of possible transfer, but after receiving the specific training works all thirty-two correctly, the transfer would then be $3\frac{1}{2}\%$ or 100 per cent of the maximum possible transfer. If, on the other hand, he works only sixteen of the thirty-two, the transfer would be only $1\frac{1}{2}\%$, or 50 per cent complete. Or, to take an actual case, pupil 2-B worked two examples in the region of possible transfer on the first testing. The greatest possible transfer for her was, therefore, not 32, but 32 minus 2, or 30. She worked fifteen examples in this region, on the second testing; but, since she worked two of these on the first testing, this represented a spread to only 15 minus 2, or to 13 new examples. The transfer was, therefore, $1\frac{1}{3}\%$ or 43.3 per cent complete. It should be noted that Type 29, carrying in addition, was omitted in figuring the percentage of transfer, since the two examples of this type did not appear on the tests given to part of the pupils.

The total percentage of transfer for each type of example is given in Table IX. Types 5 and 6 were taught after the first testing, Types 7 and 8 after the second, and Type 9 after the third. No figures are given, therefore, for Types 5 and 6; and those given for Types 7 and 8 represent the percentage of transfer on the second testing; those for Type 9 are for the third testing; and those for Types 10 to 29 are for the fourth testing.

The percentages given in Table IX were figured as follows: On the first testing, the 448 pupils worked correctly 316 of the 896 examples of Type 10, and on the fourth testing they worked 786 correctly. They, therefore, made a gain of 786 minus 316, or 470 examples, out of a possible gain of 896 minus 316, or 580 examples.

The transfer was, therefore, $47\frac{9}{580}$ or 81.0 per cent complete. The data of Table IX are taken from Table LVI, in the Appendix.

TABLE IX.—TOTAL PERCENTAGE OF TRANSFER ON EACH TYPE OF EXAMPLE

Type number	Percent- age of transfer	Testing	Type number	Percent- age of transfer	Testing
7	64.3	Second	19	73.1	Fourth
8	66.3	Second	20	68.2	Fourth
9	42.1	Third	21	54.7	Fourth
10	81.0	Fourth	22	44.8	Fourth
11	89.8	Fourth	23	61.7	Fourth
12	77.9	Fourth	24	54.2	Fourth
13	83.4	Fourth	25	44.6	Fourth
14	76.1	Fourth	26	57.2	Fourth
15	77.4	Fourth	27	46.2	Fourth
16	75.5	Fourth	29	54.1	Fourth
17	75.7	Fourth	29	2.9	Fourth
18	90.4	Fourth	All types combined	65.7	

II. COMPARISON OF METHODS OF TEACHING

The major purpose of this study was to determine whether it is possible to increase the percentage of transfer by the method of teaching. Specifically, the problem was to determine if the transfer could be increased by generalization, rationalization, and generalization and rationalization combined. It will be recalled that Method A employed neither generalization nor rationalization, Method B generalization only, Method C rationalization only, and Method D both generalization and rationalization.

1. *Comparison of Effectiveness with Respect to Types Taught.*—Before comparing the transfer obtained by the different methods of teaching, it is necessary to determine whether there were any substantial differences between the methods with respect to the types of examples that were taught during the experiment. If any of the methods produced greater accuracy than the others, on the types taught, we would naturally expect it to produce greater transfer to the related types. Table X gives

TABLE X.—EFFECTIVENESS OF THE DIFFERENT METHODS OF TEACHING WITH RESPECT TO THE TYPES OF EXAMPLES THAT WERE TAUGHT. PERCENTAGE OF EXAMPLES CORRECT ON FOURTH TESTING

Type number	Method of instruction			
	A	B	C	D
5	96.9	99.1	93.8	96.4
6	93.8	100.0	93.8	98.2
7	95.1	92.9	92.0	92.4
8	93.8	97.3	93.8	96.4
9	92.9	94.6	92.9	92.9
Mean.....	94.5	96.8	93.3	95.3

the percentages of accuracy, on the fourth testing, for each of the types taught (Types 5 to 9), and for each method of teaching.

The data of Table X indicate that there were no significant differences between the different methods as far as obtaining results on the specific types taught was concerned. Any differences in the transfer obtained by the different methods, therefore, can not be due to this cause.

2. *Comparison of Mean Transfer.*—Table XI gives the mean transfer, the standard deviation and the standard deviation of the mean, on the fourth testing, for each

TABLE XI.—PERCENTAGE OF TRANSFER OBTAINED BY DIFFERENT METHODS OF INSTRUCTION

	Method of instruction			
	A	B	C	D
Mean.....	59.6	72.4	62.8	71.8
S. D.....	27.0	25.1	26.4	23.2
S. D. of mean.....	2.55	2.37	2.49	2.19

of the different methods. The data of this table are taken from Table LII, in the Appendix, which gives the complete distributions.

The data of Table XI indicate that there were substantial differences between the amounts of transfer obtained by the different methods of teaching. In order to facilitate a further study of these differences, Table XII gives a comparison of each of the other methods to Method A. The differences in this table were obtained by subtracting the mean transfer for Method A from the means for each of the other methods. The most important facts shown by Table XII are (1) the generalization, as employed in Method B, increased the amount of transfer by 21.5 per cent; (2) the rationalization, as employed in Method C, increased the transfer by only 5.4 per cent; and (3) the generalization and rationalization combined, as employed in Method D, increased the transfer by 20.5 per cent, slightly less than the generalization alone.

In order that we may get a measure of the reliability of these comparisons, Table XII also gives the standard

TABLE XII.—COMPARISON OF EACH OF THE OTHER METHODS OF INSTRUCTION TO METHOD A

	B-A	C-A	D-A
Difference of means.....	12.8	3.2	12.2
S. D. of difference.....	3.48	3.56	3.36
Difference of means S. D. of difference	3.67	.90	3.63
Percentage of increase over Method A..	21.5	5.4	20.5

deviations of the differences and the ratios of the differences to their standard deviations. If the ratio of a difference to its standard deviation is equal to 1, the chances are 2.15 to 1 that the difference is significant; if the ratio is equal to 2, the chances are 21 to 1; and if it equals 3 the chances are 369 to 1. Any difference equal to three times its standard deviation is practically certain, therefore, to be significant.¹ According to this, the data of Table XII show that Methods B and D are almost certainly more effective than Method A in producing transfer, but that there is probably no significant difference between Method C and Method A.

The significance of the differences between Method A and each of the other methods of teaching is shown graphically in Figure 2. The bars represent the differences of the means, expressed in terms of their standard deviations. In the cases of Method B and Method D, the bars lie in the zone of "certain differences"; but, in the case of Method C, the bar lies in the zone of "probably no difference."

¹ McCall, W. A. *How to Measure in Education*, 406. New York; Macmillan, 1922.

3. *Comparison of Transfer for Groups as Wholes.*—In Table XII the amounts of transfer obtained by the different methods of teaching were studied by comparing

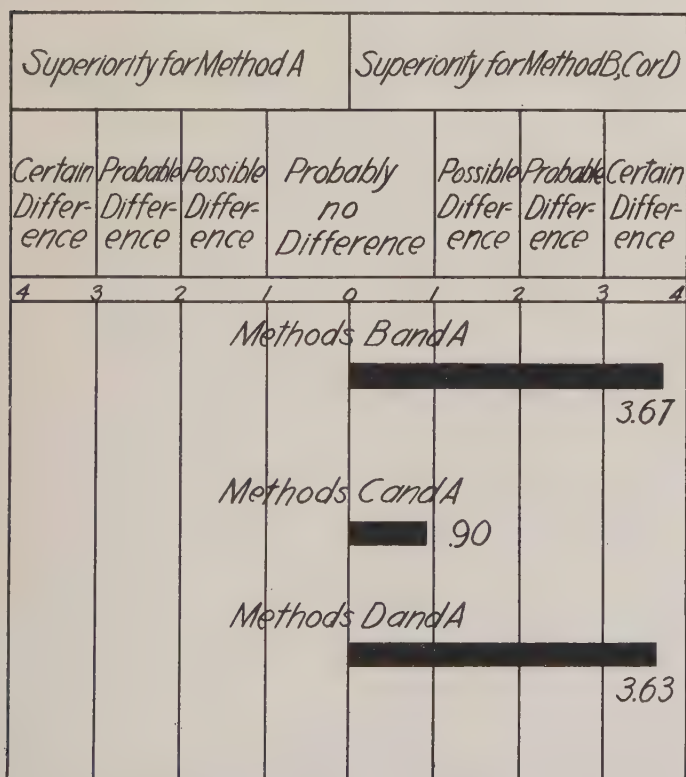


FIG. 2.—The significance of the differences between Method A and each of the other methods, expressed in terms of the standard deviations of the differences.

the means of the groups taught by these methods. Table XIII makes a similar comparison on a different basis. This table shows the percentages of transfer for each

group as a whole. For example, the 112 pupils taught by Method A worked 755 out of a possible 3584 examples, of Types 7 to 28, on the first testing. On the last testing, they worked 2461 correctly. There was, therefore, an increase of 1686 examples out of a possible increase of 2809, or an increase of 60.0 per cent of the possible increase. In the case of Types 7, 8 and 9, the score on the last testing before these types were taught was used, instead of the score on the final testing. The data for this table are taken from Table LVI, in the Appendix.

TABLE XIII.—COMPARISON OF TRANSFER OF GROUPS TAUGHT BY DIFFERENT METHODS, BASED ON PERCENTAGES FOR GROUPS AS WHOLE

	Method of instruction			
	A	B	C	D
Percentage of transfer of group as a whole.....	60.0	71.8	60.7	70.4
Percentage of increase over Method A.....		19.7	1.2	17.3

The results obtained by this method of comparison do not differ greatly from those obtained by comparing the means. In general the percentages of increase of transfer obtained by Methods B, C and D, as compared to Method A, are a little smaller.

4. *Transfer to Different Types of Examples.*—So far, in comparing the different methods of teaching, we have considered the transfer for all of the different types of examples combined. There is reason to suppose, however, that the differences in the methods will be greater for some types than for others. The examples of Types 18 to 28 all involve the placing of numbers having differ-

ent numbers of digits; and it is to these types that we would naturally expect the generalizations and the rationalizations used in Methods B, C and D to be most effective in producing transfer. Types 7, 8, 10, 11, 12, 13, 14, 15, 16, and 17 do not involve this difficulty of placing numbers having different numbers of digits and there is little reason to suppose that Methods B, C and D will be much more effective than Method A in producing transfer to these types. Since Type 9 was taught before the fourth testing, the transfer to this type was measured on the third testing, before the pupils were given any instruction on placing addends having different numbers of digits. As a result, the differences in the instruction given the different groups did not have an opportunity to affect the transfer to this type. Type 9 is, therefore, included with the group for which we would expect little difference between the methods of teaching.

Table XIV gives the means, the standard deviations, and the standard deviations of the means for the transfer to Types 7 to 17. The data of this table are taken from

TABLE XIV.—PERCENTAGE OF TRANSFER TO TYPES 7 TO 17
OBTAINED BY DIFFERENT METHODS OF TEACHING

	Method of instruction			
	A	B	C	D
Mean.....	73.2	72.6	67.2	77.1
S. D.....	30.1	30.7	27.1	30.8
S. D. of mean.....	2.85	2.90	2.56	2.91

Table LIII, in the Appendix, which gives the full distributions. An examination of Table XIV shows that the

differences between the various methods were not very great for these types.

In order to bring this out more clearly, Table XV compares each of the other methods of instruction to Method A. The small amount of the differences, and

TABLE XV.—COMPARISON OF EACH OF THE OTHER METHODS OF INSTRUCTION TO METHOD A FOR TYPES 7 TO 17

	B-A	C-A	D-A
Difference of means.....	-0.6	-6.0	3.9
S. D. of difference.....	4.07	3.83	4.07
Difference of means S. D. of difference	0.15	1.57	0.96

the small ratios of the differences to their standard deviations, indicate that the differences are probably not significant.

The significance of the differences is shown graphically in Figure 3. None of the bars extend into the zone of "probable difference" and only one into the zone of "possible difference." We can, therefore, conclude that the data show no significant differences between the methods, in ability to produce transfer to examples of Types 7 to 17.

Table XVI gives the means, the standard deviations, and the standard deviations of the means for the transfer to Types 18 to 28. The data of this table are taken from Table LIV, in the Appendix, which gives the complete distributions. A study of Table XVI shows that there were substantial differences between the methods of teaching in producing transfer to Type 18 to 28.

Each of the other methods of teaching is compared to Method A in Table XVII. The data of this table show that, for the examples of Types 18 to 28, the

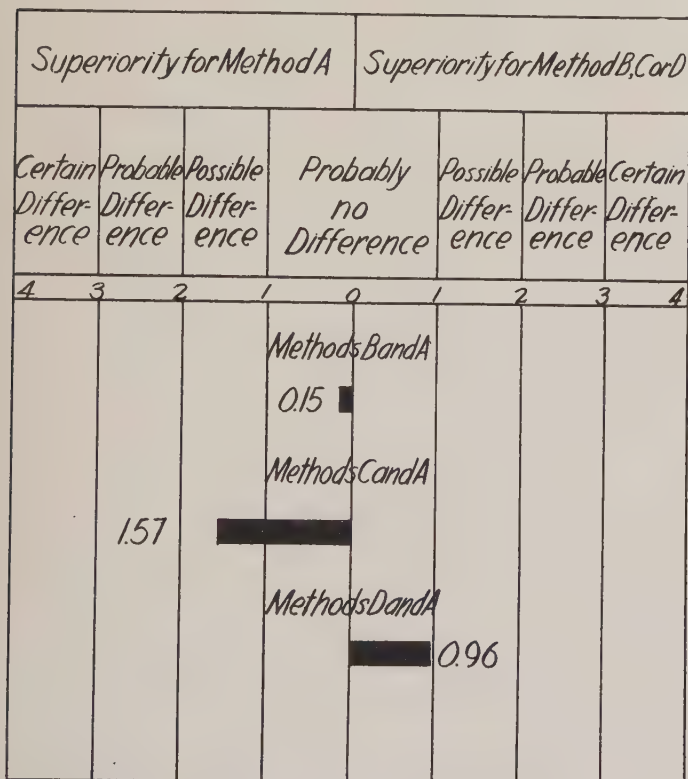


FIG. 3—The significance of the differences between Method A and each of the other methods, for Types 7 to 17, expressed in terms of the standard deviations of the differences.

generalization, as employed in Method B, increased the amount of transfer by 45.1 per cent; the rationalization, as employed in Method C, increased it by 15.5 per cent;

and the combined generalization and rationalization, as employed in Method D, increased it by 36.9 per cent. The size of the differences, and of the ratios of the

TABLE XVI.—PERCENTAGE OF TRANSFER TO TYPES 18 TO 28,
OBTAINED BY DIFFERENT METHODS OF TEACHING

	Method of instruction			
	A	B	C	D
Mean.....	46.6	67.6	53.8	63.8
S. D.....	35.6	33.4	35.6	32.7
S. D. of mean.....	3.37	3.16	3.37	3.09

differences to their standard deviations, indicate that Methods B and D are almost certainly superior to Method A in producing transfer to Types 18 to 28; and that Method C is possibly superior.

TABLE XVII.—COMPARISON OF EACH OF THE OTHER METHODS OF
INSTRUCTION TO METHOD A FOR TYPES 18 TO 28

	B-A	C-A	D-A
Difference of means.....	21.0	7.2	17.2
S. D. of difference.....	4.62	4.77	4.57
Difference of means S. D. of difference	4.54	1.51	3.76
Percentage of increase over Method A.....	45.1	15.5	36.9

The significance of the differences shown in Table XVII is represented graphically in Figure 4. The bars extend into the zone of "certain difference," in the case of

Methods B and D; and into the zone of "possible difference," in the case of Method C.

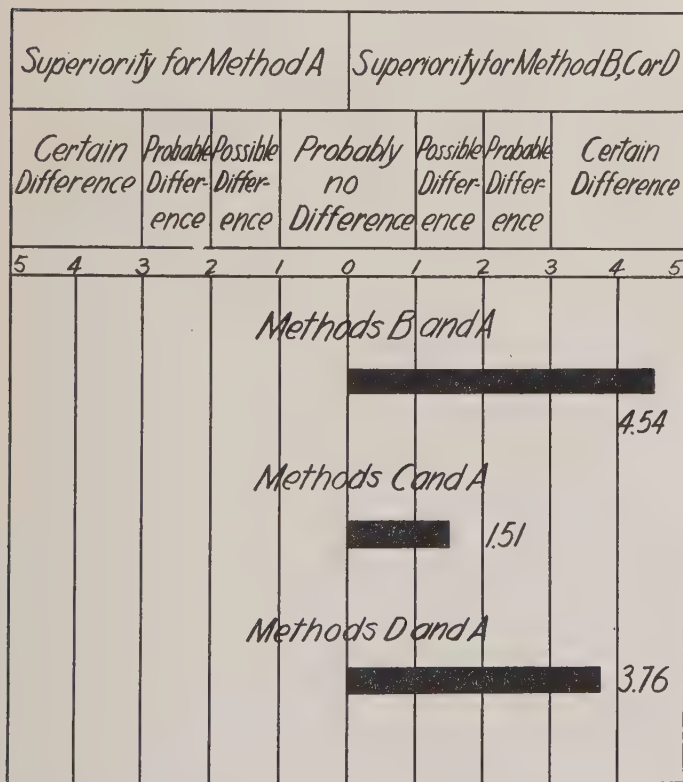


FIG. 4.—The significance of the differences between Method A and each of the other methods, for Types 18 to 28, expressed in terms of the standard deviations of the differences.

As a further check on the reliability of the differences between the different methods, an investigation was made to determine on how many of the different types of examples the superiorities shown were manifested.

Table XVIII gives the percentages of transfer for each type of example. These were taken from Table LVI, in the Appendix.

TABLE XVIII.—PERCENTAGE OF TRANSFER FOR EACH METHOD OF INSTRUCTION TO EACH TYPE OF EXAMPLE

Type number	Method of instruction			
	A	B	C	D
7	72.8	69.5	47.8	66.7
8	62.0	78.8	57.1	70.2
9	47.3	50.6	30.1	41.3
10	79.1	83.1	76.2	85.6
11	87.8	92.9	88.5	90.3
12	72.4	73.4	74.2	91.7
13	83.6	80.4	83.5	85.9
14	78.0	75.3	71.8	79.3
15	75.4	81.8	69.5	83.2
16	76.5	82.8	66.7	77.1
17	73.0	81.8	69.7	78.8
18	86.9	92.7	91.5	94.8
19	71.0	75.0	68.5	77.8
20	59.8	74.0	65.0	73.6
21	37.4	64.8	55.1	61.3
22	32.1	58.2	34.0	53.6
23	47.6	70.7	56.3	72.6
24	38.7	65.1	55.0	57.4
25	33.9	60.0	44.5	39.8
26	45.9	69.7	46.6	64.1
27	35.9	55.7	42.7	50.7
28	38.6	64.2	50.0	63.3

In order to make it easier to grasp the essential facts of Table XVIII, the signs of the differences obtained by subtracting the percentage of transfer of Method A from that of each of the other methods are given in

Table XIX. A negative sign in this table, therefore, indicates superiority for Method A. An examination of Table XIX shows that, in the case of Types 7 to 17, Method B was superior to Method A on eight of the eleven types, Method C was inferior to Method A on nine of the eleven, and Method D was superior to Method A on nine of the eleven. This may indicate that

TABLE XIX.—COMPARISON OF EACH OF THE OTHER METHODS WITH METHOD A, FOR EACH TYPE OF EXAMPLE

Type number	B-A	C-A	D-A	Type number	B-A	C-A	D-A
7	- ^a	-	-	18	+	+	+
8	+	-	+	19	+	-	+
9	+	-	-	20	+	+	+
10	+	-	+	21	+	+	+
11	+	+	+	22	+	+	+
12	+	+	+	23	+	+	+
13	-	-	+	24	+	+	+
14	-	-	+	25	+	+	+
15	+	-	+	26	+	+	+
16	+	-	+	27	+	+	+
17	+	-	+	28	+	+	+

^a A negative sign indicates superiority for Method A.

Methods B and D were slightly more effective, and Method C slightly less effective than Method A, on Types 7 to 17, in spite of the small differences shown in Table XV.

In the case of Types 18 to 28, Methods B and D were superior to Method A on all eleven of the types, and Method C was superior on ten of them. This confirms the evidence previously given that Methods B and D were almost certainly superior to Method A, in produc-

ing transfer to Types 18 to 28; and considerably strengthens the evidence that Method C was somewhat superior.

5. *Transfer Obtained by Individual Pupils.*—The amount of transfer obtained by individual pupils ranged all the way from none to complete transfer, that is, the correct solution of all of the untaught types. Of the 448 pupils only four, or 0.9 per cent obtained no transfer whatsoever. That is, to state it positively, 99.1 per cent of the pupils showed more or less transfer. Six pupils taught by Method A, or 5.4 per cent; seventeen taught by Method B, or 15.2 per cent; eleven taught by Method C, or 9.8 per cent; and twelve taught by Method D, or 10.7 per cent, obtained 100 per cent transfer. Altogether forty-six of the 448 pupils, or 10.3% showed complete transfer. The complete distributions of the transfer for each of the four methods are given in Table LII, in the Appendix.

6. *Summary.*—The data presented seem to warrant the following conclusions with respect to the effectiveness of the different methods of instruction in producing transfer.

1. The four methods were about equally effective in producing transfer to those types of examples which did not involve the difficulty of placing numbers having different numbers of digits (Types 7 to 17).

2. Methods B and D were almost certainly, and Method C was possibly more effective than Method A in producing transfer to those types of examples involving the placing of numbers having different numbers of digits (Types 18 to 28). The generalization alone (Method B) increased the transfer by 45.1 per cent, the rationalization alone (Method C) increased it 15.5 per cent, and the two combined (Method D) increased it 36.9 per cent. Aid in generalization, therefore, would seem to be the most effective method of

increasing transfer with pupils of this age. These results seem to be most logically explained as follows:

1. The examples of Types 7, 8, 10, 11, 12, 13, 14, 15, 16 and 17 were apparently so closely related to the specific types that were taught that the transfer took place automatically, without any conscious generalization on the part of the pupils, or else the generalization was so simple and obvious that the majority of the pupils were able to make it without assistance.

2. The connection between Types 18 to 28 and the only type taught that involved placing numbers having different numbers of digits (Type 9) was apparently neither so close nor so obvious. The result was that the transfer did not take place automatically and many of the pupils were unable to make the generalization without assistance. The majority were able, however, to form the generalization with the assistance of the teacher and the class discussion.

3. The fact that such large differences in the amount of transfer existed between the different groups is one of the strongest bits of evidence that the increase in scores, on types of examples in which no instruction or practice was given during the experiment, was really due to transfer from the instruction and practice given on related types, and not due to any outside instruction. It is not reasonable to suppose that there were any large differences in whatever outside instruction the groups may have had. The differences in the results must, therefore, be due to the differences in the instruction given in school during the course of the experiment.

III. COMPARISON OF SEXES

One of the questions naturally arising in connection with this study is the question as to whether there were

any marked differences between the two sexes with respect to the amount of transfer. Before making a comparison of transfer it is necessary to see how well the two sexes were matched.

1. *The Matching of the Sexes.*—Table XX gives a summary of the mental ages, initial scores, teachers' estimates and chronological ages for each of the sexes. The data of this table are taken from Tables XLV to XLVIII, in the Appendix, which give the complete distributions. The data of Table XX show that the mean mental age of the girls was, for each group, slightly greater than that of the boys, and that the two sexes were rather closely matched with respect to the other traits.

TABLE XX.—COMPARISON OF THE SEXES WITH RESPECT TO MENTAL AGE, INITIAL SCORE, TEACHERS' ESTIMATE AND CHRONOLOGICAL AGE

		Method A		Method B		Method C		Method D		All Methods	
		Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys
Mental age	Mean.....	101.5	99.6	101.6	99.7	101.7	100.0	101.7	99.9	101.6	99.8
	S. D.....	11.0	9.3	10.8	9.7	10.9	9.4	10.7	9.5	10.8	9.5
	Range....	52	39	51	45	53	43	53	40	55	45
Initial score	Mean.....	25.6	25.7	27.9	28.0	26.7	25.9	26.7	25.7	26.7	26.3
	S. D.....	9.8	9.3	8.9	8.5	9.4	9.6	9.9	8.8	9.5	9.1
	Range....	37	35	38	30	35	39	36	33	38	39
Teachers' estimate	Mean.....	2.7	2.7	2.5	2.4	2.5	2.3	2.6	2.4	2.6	2.4
	S. D.....	10.1	9.0	7.2	9.0	9.6	8.3	8.6	7.3	8.9	8.5
	Range....	3	4	3	3	4	4	4	4	4	4
Chronological Age	Mean.....	96.3	96.2	97.5	96.9	96.2	97.1	95.9	95.6	96.3	96.5
	S. D.....	6.9	6.4	6.3	6.7	7.1	6.7	6.9	6.8	6.8	6.7
	Range....	26	28	28	26	28	26	29	29	30	29

2. *Comparison of Mean Transfer.*—Table XXI gives the mean transfer, the standard deviation, and the stand-

ard deviation of the mean for each of the sexes and each method of instruction. The data of this table are taken from Table LII, in the Appendix. Table XXI shows that there were no large differences in transfer between the two sexes.

TABLE XXI.—PERCENTAGE OF TRANSFER FOR EACH OF THE SEXES

	Method A		Method B		Method C		Method D		All Methods	
	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys
Mean.....	58.8	60.3	70.0	74.6	62.0	63.6	70.0	73.5	65.2	68.0
S. D.....	28.2	26.0	24.4	25.5	25.9	26.8	26.6	20.3	26.8	25.5
S. D. of mean..	3.87	3.39	3.35	3.32	3.56	3.49	3.65	2.64	1.84	1.66

A more detailed comparison of the two sexes is made in Table XXII. The differences in this table were obtained by subtracting the mean transfer for the girls from that for the boys. It should be noted that, although the differences are small they are all positive,

TABLE XXII.—COMPARISON OF TRANSFER OBTAINED BY THE TWO SEXES

	Method A	Method B	Method C	Method D	All Methods
Difference of means.....	1.5	4.6	1.6	3.5	2.8
S. D. of difference	5.14	4.72	4.99	4.50	2.48
$\frac{\text{Difference}}{\text{S. D. of difference}}$	.29	.97	.32	.78	1.13
Percentage of superiority of boys over girls	2.6	6.6	2.6	5.0	4.3

that is in favor of the boys; in spite of the fact that the boys, as a group, were somewhat inferior to the girls. The ratio of the difference of the means to the standard

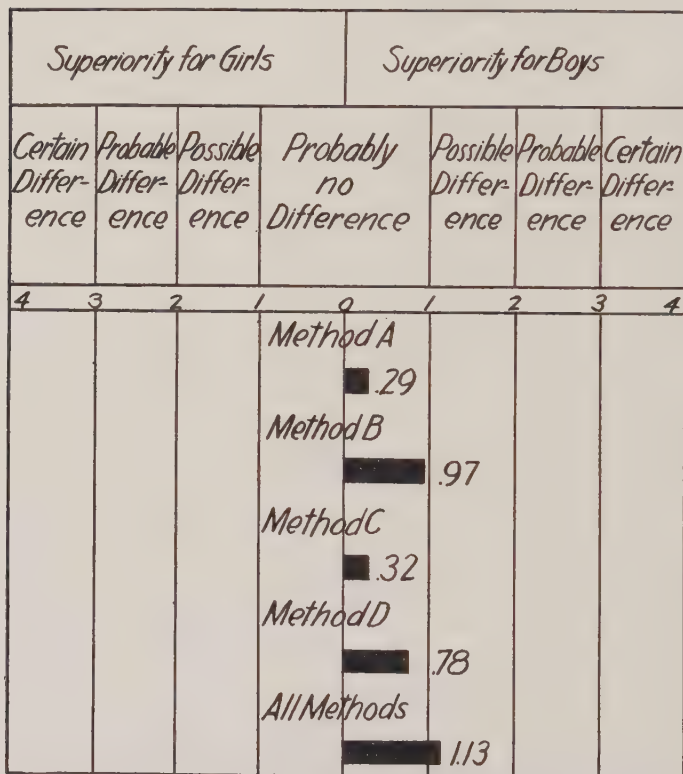


FIG. 5.—The significance of the differences in transfer between the sexes, expressed in terms of the standard deviations of the differences.

deviation of the difference is less than one for each of the methods of teaching, indicating that the differences found are probably not very reliable. The comparison is somewhat more reliable when the four methods of

teaching are combined, due to the increased size of the groups.

The significance of the differences between the sexes is shown graphically in Figure 5. None of the bars enter the zone of "probable difference." It seems safe to conclude that there were probably no significant differences in transfer between the boys and girls taught by Methods A and C, but that the boys taught by Methods B and D may possibly have shown slightly more transfer than the girls taught by those methods. The slight differences found in favor of the boys become somewhat more significant when we recall that the mean mental age of the boys was somewhat below that of the girls.

3. *Comparison of Transfer for Groups As Wholes.*—The percentages of transfer for the boys as a group and for the girls as a group were also figured. For example, the fifty-three girls who were taught by Method A, worked 373 out of a possible 1696 examples, of Types 7 to 28, on the first testing. On the last testing¹ they worked 1171 correctly. There was, therefore, an increase of 798 examples out of a possible increase of 1323, or an increase of 60.3 per cent of the possible increase. Table XXIII compares the two sexes on this basis. The data of this table are taken from Table LVI, in the Appendix.

The comparison of the sexes made in Table XXIII shows results quite similar to those of Table XXI, which made a similar comparison based on the means. The most important difference is that, while the comparison of the means showed the boys taught by Method A to be slightly superior to the girls taught by this method, the comparison of these groups as wholes shows the girls to be somewhat superior. This strengthens the former

¹ The scores for Types 7, 8 and 9 were taken on the last testing before these types were taught, the others on the fourth testing.

conclusion that there was probably no significant difference between the sexes when taught by Method A. The percentages of superiority for the boys taught by the other methods are somewhat larger in Table XXIII than in Table XXII, which increases somewhat the possibility that these differences may be significant.

TABLE XXIII.—COMPARISON OF THE PERCENTAGES OF TRANSFER FOR THE TWO SEXES, FIGURED FOR THE GROUPS AS WHOLE

	Method A		Method B		Method C		Method D		All Methods	
	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys
Percentage of transfer.....	60.3	59.8	68.3	75.0	59.0	62.1	68.3	72.2	63.9	67.2
Superiority of boys.....		0.5		6.7		3.1		3.9		3.3
Percentage of superiority of boys.....		-0.8		9.8		5.3		5.7		5.2

4. *Transfer to Different Types of Examples.*—In order to test further the reliability of the differences between the sexes, part of Table LVI, in the Appendix, is repeated in Table XXIV, which gives the percentage of transfer obtained by the boys and by the girls, on each type of example and for each method of teaching.

The essential facts of Table XXIV are summarized in Table XXV which compares the two sexes by giving the sign of the difference obtained by subtracting the amount of transfer for the girls from that for the boys. A positive sign in this table, therefore, indicates superiority for the boys.

A careful examination of Table XXV reveals the following rather interesting facts: (1) When taught by Method A, the girls obtained more transfer than the boys

on fifteen of the twenty-two types. They were superior on all eleven of the types from Type 18 to Type 28. (2) In the case of Method B, the boys exceeded the girls

TABLE XXIV.—PERCENTAGE OF TRANSFER FOR EACH OF THE SEXES ON EACH TYPE OF EXAMPLE

Type number	Girls					Boys				
	Method of instruction					Method of instruction				
	A	B	C	D	All	A	B	C	D	All
7	68.2	61.7	42.4	65.1	59.7	76.5	76.1	51.9	67.9	68.1
8	63.9	86.4	48.1	64.0	64.5	60.0	73.3	63.9	75.0	67.7
9	36.9	50.0	26.0	34.1	37.6	54.2	51.2	34.0	47.9	46.8
10	78.0	81.0	73.4	85.4	79.5	80.0	84.9	78.3	85.9	82.2
11	87.6	90.9	86.4	88.1	88.3	88.0	94.6	90.0	91.8	91.0
12	79.1	73.3	70.0	89.7	78.0	65.9	73.5	77.8	93.0	77.7
13	86.5	72.6	76.4	84.6	81.5	81.3	82.0	89.5	86.8	85.0
14	77.0	73.4	62.4	74.4	71.6	78.0	76.9	80.3	83.0	79.8
15	75.0	76.9	67.6	88.1	77.2	75.7	86.2	70.9	79.5	77.5
16	78.8	76.7	68.8	72.0	74.2	74.3	89.2	65.1	80.0	76.4
17	69.8	76.2	71.6	78.2	73.9	75.6	87.7	68.2	79.2	77.1
18	87.2	97.5	93.7	97.9	94.2	86.5	88.1	89.2	91.6	88.8
19	72.3	68.9	64.3	83.9	70.0	69.8	80.4	72.0	81.2	75.8
20	60.4	68.1	63.3	64.7	64.1	59.2	79.2	66.7	81.8	72.1
21	45.1	60.8	52.0	59.6	54.4	30.4	68.4	57.9	62.7	55.0
22	34.6	57.7	38.0	56.1	46.6	29.8	58.6	30.0	51.7	43.0
23	49.0	61.4	58.8	70.0	59.7	46.3	78.2	53.9	75.0	63.6
24	42.3	62.7	48.1	56.6	52.4	35.2	67.2	64.9	58.1	55.8
25	37.2	56.6	50.0	42.3	46.6	31.0	63.2	39.7	37.5	42.8
26	46.2	62.5	53.3	63.5	56.5	45.7	75.7	39.3	64.6	57.9
27	36.8	51.9	43.8	55.8	47.0	35.0	59.0	41.6	45.9	45.4
28	41.9	60.0	52.9	56.3	52.7	35.2	67.8	46.9	70.1	55.5

on twenty of the twenty-two types. (3) In the case of Method C, the boys exceeded the girls on thirteen of the twenty-two types. (4) When taught by Method D, the

boys obtained more transfer than the girls on sixteen of the twenty-two types.

5. *Summary.*—The data presented concerning the differences between the sexes, with respect to the amount of transfer obtained, seem to warrant the following conclusions.

TABLE XXV.—COMPARISON OF THE SEXES WITH RESPECT TO TRANSFER FOR EACH TYPE OF EXAMPLE

Type num- ber	Sign of difference, boys minus girls					Type num- ber	Sign of difference, boys minus girls				
	Method of instruction						Method of instruction				
	A	B	C	D	All		A	B	C	D	All
7	+	+	+	+	+	18	-	-	-	-	-
8	-	-	+	+	+	19	-	+	+	-	+
9	+	+	+	+	+	20	-	+	+	+	+
10	+	+	+	+	+	21	-	+	+	+	+
11	+	+	+	+	+	22	-	+	-	-	-
12	-	+	+	+	-	23	-	+	-	+	+
13	-	+	+	+	+	24	-	+	+	+	+
14	+	+	+	+	+	25	-	+	-	-	-
15	+	+	+	-	+	26	-	+	-	+	+
16	-	+	-	+	+	27	-	+	-	-	-
17	+	+	-	+	+	28	-	+	-	+	+

1. There is no reliable evidence of any significant differences between the sexes when taught by Method A.

2. There is some evidence, although not conclusive, that the boys taught by Methods B, C and D showed somewhat more transfer than the girls taught by these methods. The evidence is strongest in the case of Method B and weakest in that of Method C. Appar-

ently the boys profited somewhat more by the generalization than did the girls.

IV. RELATION BETWEEN TRANSFER AND INTELLIGENCE

In order to study the effect of intelligence on the percentage of transfer, the groups taught by the different methods were each divided into smaller groups, on three levels of intelligence and as nearly equal in number as possible. In making this division the matched quartets were not disturbed so that each boy or girl in any group still has a "match" in each of the corresponding groups, taught by the other three methods.

1. *Data Concerning the Three Levels of Intelligence.*—Table XXVI gives the mean mental ages and the range of

TABLE XXVI.—MEAN MENTAL AGES AND RANGE OF MENTAL AGES FOR PUPILS ON THREE LEVELS OF INTELLIGENCE

		Group				
		A	B	C	D	All
Level I	Number of pupils	37	37	37	37	148
	Mean M. A.	89.5	89.7	90.0	89.8	89.8
	Range	77-98	75-98	77-97	78-98	75-98
Level II	Number of pupils	37	37	37	37	148
	Mean M. A.	100.5	100.4	100.6	100.8	100.5
	Range	95-104	96-104	97-105	96-105	95-105
Level III	Number of pupils	38	38	38	38	152
	Mean M. A.	111.3	111.5	111.5	111.4	111.4
	Range	104-129	102-131	103-131	102-132	102-132

mental ages for each of the groups on these three levels of intelligence. In Table XXVI, and in the following discussion, Level I is the level of lowest intelligence.

Table XXVI shows that there is a slight overlapping of the mental ages on the three levels. Thus, for all the groups, the mental ages of pupils on Level I extend from 75 months to 98 months, while those on Level II begin at 95 months. This overlapping is due to the fact that the matched quartets were not disturbed. By keeping these quartets intact the groups taught by the different methods, on each level of intelligence, are still closely matched, as is shown in Table XXVI. This makes it possible to compare the effectiveness of the different methods on the various levels.

2. *Comparison of Transfer on the Three Levels.*—Table XXVII gives the mean percentage of transfer, the standard deviation, and the standard deviation of the mean for each of the three levels of intelligence. These

TABLE XXVII.—PERCENTAGE OF TRANSFER ON THREE LEVELS OF INTELLIGENCE

		Method of instruction				
		A	B	C	D	All
Level I	Mean.....	49.8	62.6	54.4	57.2	56.0
	S. D.....	26.1	26.3	25.9	25.3	26.3
	S. D. of mean.....	4.29	4.33	4.26	4.16	2.16
Level II	Mean.....	63.8	70.1	58.0	73.0	66.2
	S. D.....	25.2	25.0	25.9	20.7	24.9
	S. D. of mean.....	4.14	4.11	4.26	3.40	2.09
Level III	Mean.....	65.0	84.3	75.7	85.0	77.5
	S. D.....	27.1	18.2	22.3	14.4	22.6
	S. D. of mean.....	4.40	2.99	3.62	2.34	1.84

were taken from Table LV, in the Appendix, which gives the complete distributions. A study of Table XXVII shows that the pupils on Level III obtained more transfer than those on Level II and that these, in turn, obtained more than those on Level I; and that this was apparently true for each of the different methods of instruction.

In order to facilitate a further comparison of the different levels, Table XXVIII compares the amount of transfer on each level of intelligence to the amount on each of the lower levels. Table XXVIII shows a number

TABLE XXVIII.—COMPARISON OF THE AMOUNT OF TRANSFER ON EACH LEVEL OF INTELLIGENCE TO THE AMOUNT ON EACH LOWER LEVEL

		Method of instruction				
		A	B	C	D	All
Increase of Level II over Level I	Difference of means..	14.0	7.5	3.6	15.8	10.2
	S. D. of difference....	5.96	5.97	6.02	5.37	2.97
	Diff. ÷ S. D. of diff...	2.34	1.26	.06	2.94	3.43
	Percentage of increase	28.1	12.0	6.6	27.6	18.2
Increase of Level III over Level II	Difference of means..	1.2	14.2	17.7	12.0	11.3
	S. D. of difference....	6.04	5.06	5.59	4.13	2.75
	Diff. ÷ S. D. of diff...	.20	2.81	3.17	2.91	4.11
	Percentage of increase	1.9	20.3	30.5	16.4	17.1
Increase of Level III over Level I	Difference of means..	15.2	21.7	21.3	27.8	21.5
	S. D. of difference....	6.14	5.24	5.59	4.77	2.84
	Diff. ÷ S. D. of diff...	2.48	4.14	3.81	5.83	7.57
	Percentage of increase	30.5	34.7	39.2	48.6	38.4

of interesting facts. In the case of pupils taught by Method A, those on the second level obtained considerably more transfer than those on the lowest level, but those on the highest level obtained very little more than

automatically and does not involve the formulation of conscious generalizations by the pupils.

The rationalization employed in Method C enabled the pupils on the middle level to get very little more transfer than those on the lowest level, but enabled those on the highest level to get considerably more than those on the middle level. It would appear from this that the rationalization is particularly effective with the pupils on the highest level of intelligence. This is borne out by the large percentages of increase, of Level III over Level I, for Methods C and D.

The reliability of the differences in transfer, between the different levels of intelligence, is shown graphically in Figure 6. A glance at this figure shows that the only ones that lie in the zone of "probably no difference" are the difference between Levels II and I, for Method C; and the difference between Levels III and II, for Method A. These have already been discussed.

3. *Comparison of the Methods of Teaching on the Three Levels.*—In Table XXIX, each of the other methods of teaching is compared to Method A, on each level of intelligence. An examination of this table shows that the differences between the different methods are small, and comparatively unreliable, on Levels I and II, but quite substantial on Level III.

The significance of the differences between the different methods of teaching, on each level of intelligence, is shown graphically in Figure 7. An examination of this figure shows that none of the differences are in the zone of "certain difference" until we reach the highest level.

The data seem to warrant the following conclusions: (1) there were no certain differences between the methods for pupils on the two lower levels of intelligence, although there is some evidence to indicate that Method B was

possibly slightly superior even with these pupils. (2) Methods B and D were almost certainly more effective than Method A in producing transfer with pupils on the highest level, and Method C was possibly superior.

TABLE XXIX.—COMPARISON OF THE AMOUNT OF TRANSFER OBTAINED BY EACH OF THE OTHER METHODS OF TEACHING TO THAT OBTAINED BY METHOD A, FOR EACH OF THE THREE LEVELS OF INTELLIGENCE

		B-A	C-A	D-A
Level I	Difference of means.....	12.8	4.6	7.4
	S. D. of difference.....	6.10	6.05	5.98
	Diff. ÷ S. D. diff.....	2.10	.76	1.24
	Percentage of increase.....	25.7	9.2	14.9
Level II	Difference of means.....	6.3	-5.8	2.4
	S. D. of difference.....	5.83	5.94	5.36
	Diff. ÷ S. D. of diff.....	1.08	.98	.45
	Percentage of increase.....	6.3	5.8	2.4
Level III	Difference of means.....	19.3	10.7	20.0
	S. D. of difference.....	5.31	5.70	4.98
	Diff. ÷ S. D. of diff.....	3.61	1.88	4.02
	Percentage of increase.....	29.7	16.5	30.8

4. *Comparison of Methods of Teaching on Highest Level for Types 18 to 28.*—It has been found that the differences between the methods was greatest for examples of Types 18 to 28 and for pupils on the highest level of intelligence. This suggested a comparison of the different methods of instruction on Level III, for examples of Types 18 to 28. Table XXX gives the mean transfer, the standard deviation and the standard deviation of the mean for these groups. Table XXX not only shows substantial superiority for each of the other methods, as

The significance of the differences shown in Table XXXI is represented graphically in Figure 8. Table

TABLE XXX.—PERCENTAGE OF TRANSFER TO EXAMPLES OF TYPES 18 TO 28, FOR PUPILS ON THE HIGHEST LEVEL OF INTELLIGENCE

	Method of instruction			
	A	B	C	D
Mean.....	54.3	81.2	69.2	84.8
S. D.....	35.4	28.0	33.1	19.1
S. D. of mean.....	5.75	4.55	5.37	3.10

XXXI and Figure 8 indicate that Methods B and D were almost certainly superior to Method A, for pupils on the highest level of intelligence, in producing transfer to

TABLE XXXI.—COMPARISON OF THE AMOUNT OF TRANSFER TO EXAMPLES OF TYPES 18-28, OBTAINED BY THE DIFFERENT METHODS OF TEACHING, BY PUPILS ON THE HIGHEST LEVEL OF INTELLIGENCE

	B-A	C-A	D-A	D-B
Difference of means.....	26.9	14.9	30.5	3.6
S. D. of difference.....	7.33	7.87	6.53	5.51
Difference of means				
S. D. of difference.....	3.67	1.89	4.67	.65
Percentage of superiority..	49.5	27.4	56.2	3.9

examples of Types 18 to 28; just as they were for all the pupils. The data do not show, however, any significant

superiority of Method D over Method B. It would be interesting to try an experiment similar to the present one with pupils in grades above the second, in order to

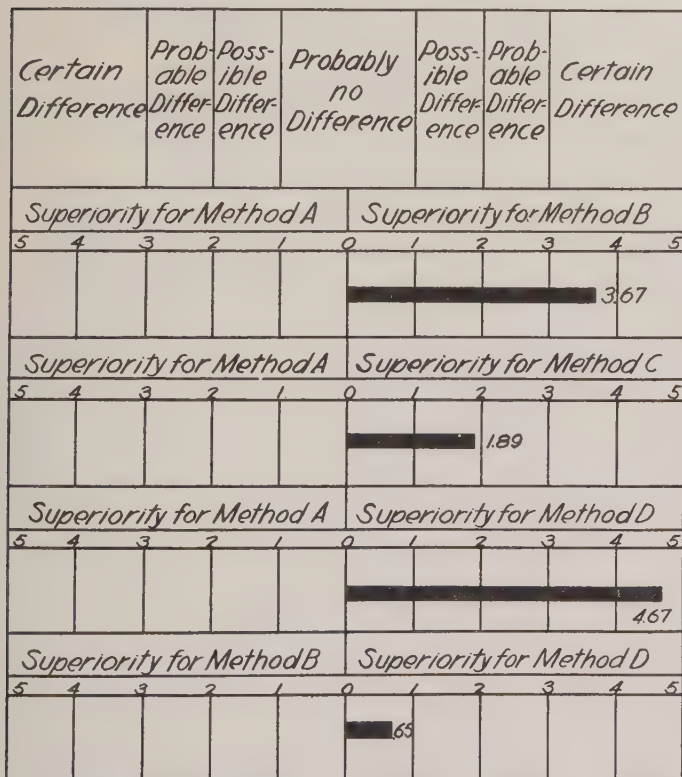


FIG. 8.—The significance of the differences in amount of transfer to examples of Types 18 to 28, obtained by pupils on the highest level of intelligence when taught by the different methods. The differences are expressed in terms of their standard deviations.

determine whether more transfer can be obtained, with older pupils, by a combination of generalization and rationalization, than by generalization alone.

5. *Correlation between Transfer and Mental Age.*—The division of the pupils into groups on three levels of intelligence showed that there was some relation between intelligence and transfer. In order to study this relation further scatter-diagrams were made (see Figures 9 to 12). Each dot represents one pupil. The crosses mark the mean transfer for each column of the scatter-diagram. These scatter-diagrams show that the relation between

TABLE XXXII.—CORRELATION BETWEEN TRANSFER AND INTELLIGENCE

Sex	Method	r	Equation of line of regression of y (transfer) on x (mental age)
B G	A	0.20 ± 0.08 0.24 ± 0.09	$y = 0.55 x$ $y = 0.62 x$
B G	B	0.46 ± 0.07 0.34 ± 0.08	$y = 1.21 x$ $y = 0.78 x$
B G	C	0.30 ± 0.08 0.43 ± 0.08	$y = 0.85 x$ $y = 1.02 x$
B G	D	0.42 ± 0.07 0.63 ± 0.06	$y = 0.91 x$ $y = 1.55 x$
Both	A	0.22 ± 0.06	$y = 0.58 x$
	B	0.39 ± 0.05	$y = 0.95 x$
	C	0.36 ± 0.06	$y = 0.93 x$
	D	0.54 ± 0.05	$y = 1.19 x$

transfer and intelligence is not very close, but they also show that the relation is apparently a linear one.

Since the relation between intelligence and transfer seemed to be linear the coefficients of correlation were computed. These are given in Table XXXII. The cor-

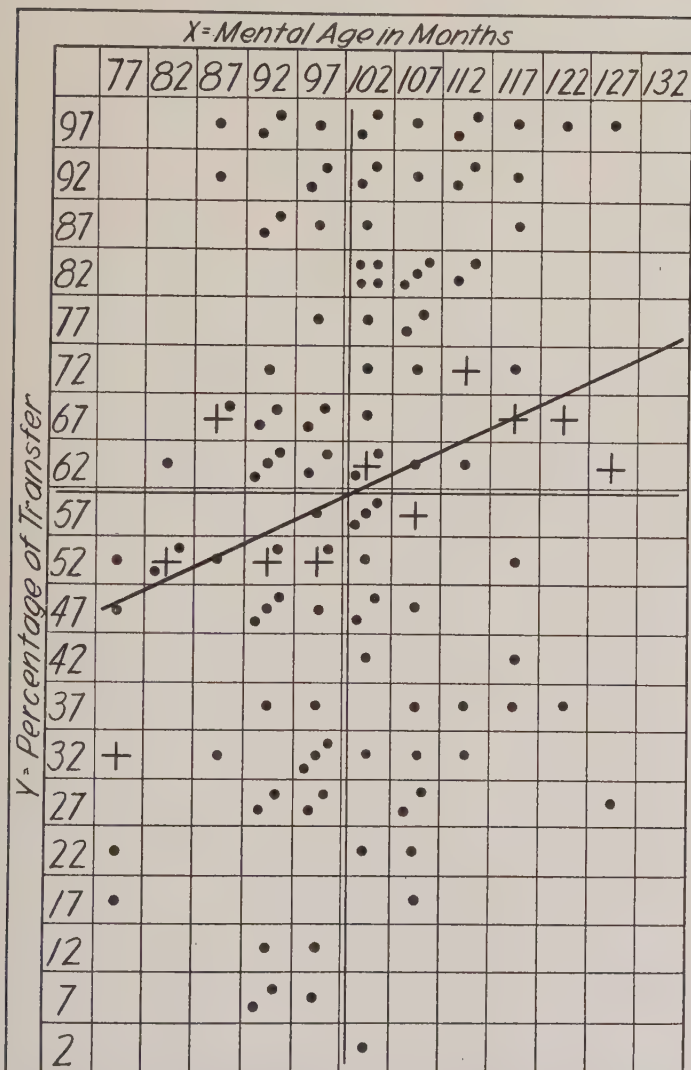


FIG. 9.—Scatter-diagram showing relation between mental age and percentage of transfer for the 112 pupils taught by method A. Line of regression of Y (transfer) on X (mental age).

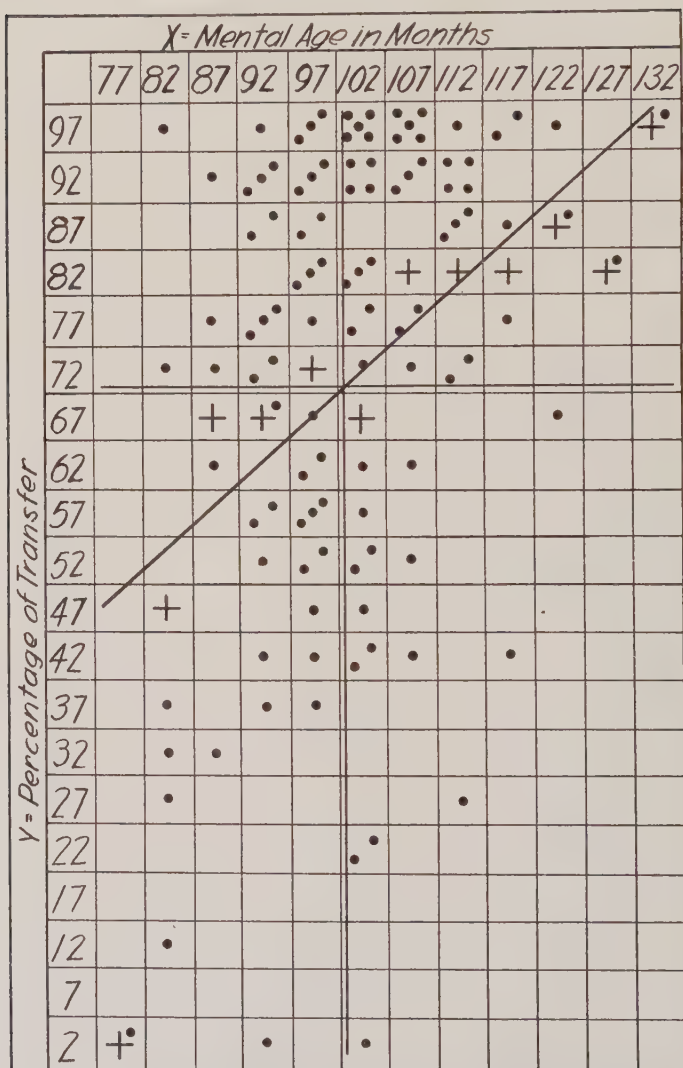


FIG. 10.—Scatter-diagram showing relation between mental age and percentage of transfer for the 112 pupils taught by method B. Line of regression of Y (transfer) on X (mental age).

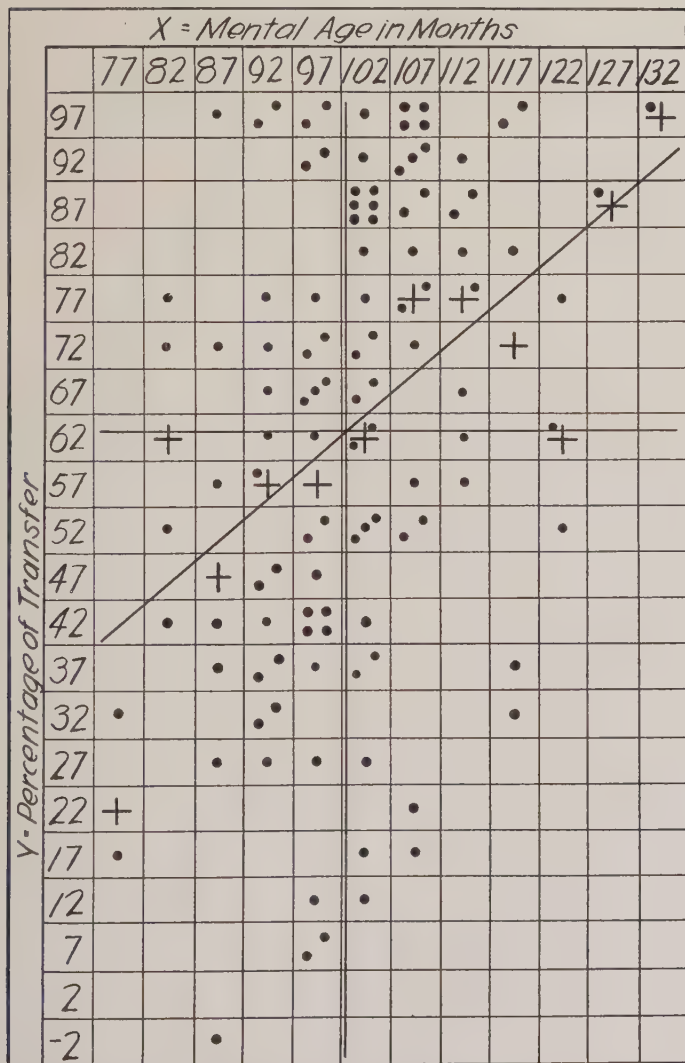


FIG. 11.—Scatter-diagram showing relation between mental age and percentage of transfer for the 112 pupils taught by method C. Line of regression of Y (transfer) on X (mental age).

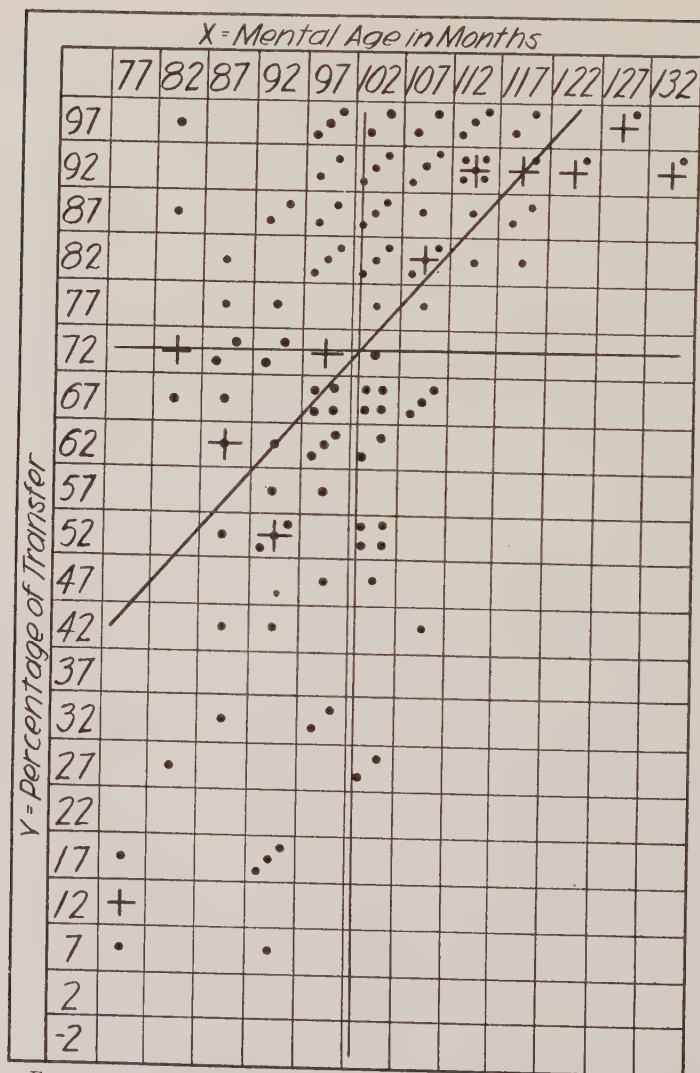


FIG. 12.—Scatter-diagram showing relation between mental age and percentage of transfer for the 112 pupils taught by method D. Line of regression of Y (transfer) on X (mental age).

relations run from 0.20 to 0.63 with Probable Errors varying from 0.05 to 0.09. Method D shows the highest correlation and Method A the lowest.

Table XXXII also gives the equations of the lines of regression of Y on X, where Y represents transfer and X represents mental age. These were computed by means of the formula¹

$$Y = r \frac{S.D.Y}{S.D.X} X.$$

The coefficients of X in these equations give the slopes of the lines of regression and therefore give a measure of the rapidity with which transfer increases, as the mental ages get larger. They show that the rise of transfer with increased mental age is most rapid for Method D, somewhat less and about equal for Methods B and C, and much less (about half as rapid as for Method D) in the case of Method A.

6. *Summary.*—The conclusions that have been drawn, from the data concerning the relationship between transfer and mental age, may be summarized as follows:

1. In general there was an increase in the amount of transfer with an increase in mental age. This was indicated both by the fact that the transfer was greater on the higher levels of intelligence than on the lower, and by the positive correlations between transfer and mental age.

2. The increase in transfer with increased mental age was greatest in the case of Method D, somewhat less and about equal for Methods B and C, and much less in the case of Method A; that is, the generalization and the rationalization were relatively more effective on the

¹ Rietz, H. L. *Handbook of Mathematical Statistics*, 126. Boston: Houghton, Mifflin Co., 1924.

higher levels than on the lower. This was shown by the greater percentages of increase of each level over the next lower and by the greater slopes of the regression lines, in the cases of Methods B, C and D.

3. No certain differences were found between the transfer produced by the different methods on the two lower levels, although there was some evidence that the generalization increased the transfer even on these levels.

4. On the highest level the generalization increased the transfer 29.7 per cent for all examples and 49.5 per cent for Types 18 to 28; the rationalization increased the transfer by 16.5 per cent for all examples and 27.4 per cent for Types 18 to 28, and the generalization and rationalization combined increased the transfer by 30.8 per cent for all examples and by 56.2 per cent for Types 18 to 28.

V. WAS THE LOSS FROM THE ORIGINAL TO THE NEW SITUATION A FUNCTION OF THE DISSIMILARITY OF THE SITUATIONS?

Knight and Setzafandt stated one of the questions to be investigated in connection with transfer of training as follows: "Is the amount of loss of skill from the original situation to the new situation a function of the dissimilarity of the new situation to the original situation?"¹ The present experiment offers some opportunity for studying this question.

Table XXXIII shows the ranks of the different types of examples, with respect to transfer, for each of the methods of instruction. Table XXXIII was compiled from Table LVI, in the Appendix. A careful study of this table shows a very substantial agreement between

¹ Knight, F. B., and Setzafandt, A. O. H. *op. cit.*, 781.

the rankings for the different methods. It has been shown that the different methods caused considerable differences in the *total amounts* of transfer, but they apparently made no significant change in the *relative amounts* to the different types of examples.

TABLE XXXIII.—RANK OF DIFFERENT TYPES OF EXAMPLES WITH RESPECT TO PERCENTAGE OF TRANSFER

Rank	Method of instruction				
	A	B	C	D	All
1	11 ^a	11	18	18	18
2	18	18	11	12	11
3	13	10	13	11	13
4	10	16	10	13	10
5	14	15-17	12	10	12
6	16	15-17	14	15	15
7	15	13	17	14	14
8	17	14	15	17	17
9	12	19	19	19	16
10	19	20	16	16	19
11	20	12	20	20	20
12	23	23	23	23	23
13	26	26	21	26	26
14	24	24	24-28	28	21
15	28	21	24-28	21	24
16	21	28	26	24	28
17	27	25	25	22	27
18	25	22	27	27	22
19	22	27	22	25	25

^a The numbers in the body of the table represent types of examples.

In order to make it somewhat easier to study, the ranking for all of the methods combined is repeated in Table XXXIV. This table also gives the percentages of transfer (from Table LVI, in the Appendix) and a brief

description of each type. As heretofore, the notation $2 + 1 + 2$ means the addition of a two digit, a one digit and a two digit number, in the order stated.

In studying Table XXXIV it should be kept in mind that the three types of examples that were taught to the

TABLE XXXIV.—RANK AND DESCRIPTION OF DIFFERENT TYPES OF EXAMPLES WITH RESPECT TO PERCENTAGE OF TRANSFER

Rank	Type of example		Percentage of transfer
	Number	Description	
1	18	$2 + 1 + 2$	90.4
2	11	$3 + 3$	89.8
3	13	$3 + 3 + 3$	83.4
4	10	$2 + 2 + 2 + 2$	81.0
5	12	$3 + 3$ (Dictated)	77.9
6	15	$2 - 2$	77.4
7	14	$3 + 3 + 3 + 3$	76.1
8	17	$3 - 3$	75.7
9	16	$2 - 2$ (Dictated)	75.5
10	19	$1 + 2 + 2$	73.1
11	20	$1 + 2 + 1$	68.2
12	23	$3 + 1$	61.7
13	26	$2 - 1$	57.2
14	21	$3 + 2$	54.7
15	24	$1 + 3$	54.2
16	28	$3 - 1$	54.1
17	27	$3 - 2$	46.2
18	22	$2 + 3$	44.8
19	25	$2 + 3 + 1$	44.6

pupils during the course of the experiment were the types $2 + 2$, $2 + 2 + 2$ and $2 + 2 + 1$. Several facts are evident from this table.

1. The subtraction examples, in each case, received less transfer than the corresponding types of addition exam-

ples. Thus, $3 - 3$ received less transfer than $3 + 3$ and $3 - 2$ less than $3 + 2$. This is exactly what we would expect, when we recall that no subtraction was taught during the experiment. In other words, the introduction of the new element of subtraction caused a reduction in the amount of the transfer.

2. There was less transfer to examples involving three place numbers than to the corresponding types involving two place numbers. For example, there was less transfer to $3 + 3 + 3 + 3$ than to $2 + 2 + 2 + 2$, and less to $3 - 3$ than to $2 - 2$. Again the introduction of an additional new element, this time an extra digit, reduced the amount of transfer.

3. Addition examples with four addends received less transfer than the corresponding types with three addends. Thus $3 + 3 + 3 + 3$ received less transfer than $3 + 3 + 3$. Once more the added new element, this time an extra addend, reduced the transfer.

4. In types involving numbers having different numbers of digits, there was less transfer when a number having fewer digits came before a number having a greater number of digits, than when the order was reversed. For example, there was less transfer to $1 + 3$ than to $3 + 1$, to $2 + 3$ than to $3 + 2$, and to $1 + 2 + 2$ than to $2 + 1 + 2$. In the specific type that was taught, namely $2 + 2 + 1$, the number of fewer digits came last; the reversal of this order, therefore, introduced an additional new element and reduced the amount of transfer.

5. In types involving numbers having different numbers of digits, there was less transfer when the number of fewer digits had two digits, than when it had one. Thus, $3 + 2$ received less transfer than $3 + 1$, $3 - 2$ less than $3 - 1$, and $2 + 3$ less than $1 + 3$. In

the specific type taught, namely $2 + 2 + 1$, the number of fewer digits had one digit; therefore, when it had two digits a new element was introduced and the transfer was reduced.

This analysis of the ranks of the different types of examples with respect to the percentage of transfer has been given to show that the order can be accounted for on the basis of the relations existing between the types of examples taught and the types that were not taught. The ranking as found was also probably partly due to differences in the intrinsic difficulty of the different types. The type $3 + 3 + 3 + 3$ probably received less transfer than the type $2 + 2 + 2 + 2$, not only because it is less closely related to the type $2 + 2 + 2$, which was taught, but also because it is probably more difficult, being longer and affording more opportunities for errors. It would be interesting to determine whether this greater intrinsic difficulty of $3 + 3 + 3 + 3$ could be overcome, and whether it could be made to receive more transfer than $2 + 2 + 2 + 2$, by teaching the pupils the types $3 + 3$ and $3 + 3 + 3$ instead of $2 + 2$ and $2 + 2 + 2$.

Summary.—The data presented concerning the relative amounts of transfer to the different types of examples seem to indicate that the ranking found was probably the result of two factors (1) the intrinsic difficulty of the untaught types and (2) the degree of similarity or dissimilarity between the untaught types and those that were taught.

VI. WHEN THE TRANSFER OCCURRED

So far we have considered the total amount of transfer as shown on the final testing. It will be recalled that the pupils were tested four times, at the beginning, at the end, and twice during the experiment. This enables us,

not only to determine the final amount of transfer, but also to study when the transfer occurred.

1. *Percentage of Transfer on Each Testing.*—Table XXXV gives the mean transfer, the standard deviation, and the standard deviation of the mean on each of the testings, after the first, and for each method of instruction. The data of this table are taken from Tables L to LII, in the Appendix, which give the full distributions.

TABLE XXXV.—PERCENTAGE OF TRANSFER ON EACH TESTING AND FOR EACH METHOD OF INSTRUCTION

		Method of instruction			
		A	B	C	D
Second testing	Mean.....	36.4	39.2	24.0	34.6
	S. D.....	28.6	29.4	27.5	26.3
	S. D. of mean.....	2.70	2.77	2.59	2.48
Third testing	Mean.....	46.1	53.1	38.7	47.8
	S. D.....	30.0	30.7	27.9	29.3
	S. D. of mean.....	2.83	2.90	2.63	2.76
Fourth testing	Mean.....	59.6	72.4	62.8	71.8
	S. D.....	27.0	25.1	26.4	23.2
	S. D. of mean.....	2.55	2.37	2.49	2.19

An examination of Table XXXV shows two things: (1) That there was a steady increase in the percentage of transfer, from the second to the fourth testings, for all methods of teaching, and (2) That differences exist on each testing, between the amounts of transfer obtained by the different methods.

In order to better study the increase of transfer for the successive testings, Table XXXVI was compiled from Table XXXV. Table XXXVI shows that while

the transfer is increasing with successive testings, for all methods of instruction, it is increasing most rapidly for Method C and least rapidly for Method A. The differences shown in Table XXXVI are almost certainly

TABLE XXXVI.—COMPARISON OF THE AMOUNT OF TRANSFER ON EACH TESTING TO THE AMOUNT ON THE PREVIOUS TESTING, FOR ALL METHODS OF TEACHING

		Method of instruction			
		A	B	C	D
Increase of third testing over second	Difference of means.....	9.7	13.9	14.7	13.2
	S. D. of difference.....	3.91	4.01	3.69	3.71
	$\frac{\text{Difference of means}}{\text{S. D. of difference}}$	2.48	3.47	3.98	3.56
	Percentage of increase.....	26.6	35.5	61.3	38.2
Increase of fourth testing over third	Difference of means.....	13.5	19.3	24.1	24.0
	S. D. of difference.....	3.81	3.75	3.62	3.52
	$\frac{\text{Difference of means}}{\text{S. D. of difference}}$	3.54	5.15	6.66	6.82
	Percentage of increase.....	29.3	36.4	62.3	50.2

significant, all but one being over three times as large as their standard deviations. This is shown graphically in Figure 13.

2. *Comparison of Methods of Teaching on Each Testing.* Table XXXVI showed that the rate of increase of transfer, for successive testings, was not the same for all methods. In order to make the comparison of the

methods easier, Table XXXVII was compiled from Table XXXV.

The data of Table XXXVII indicate that, on the second testing, Methods B and D were about equal to

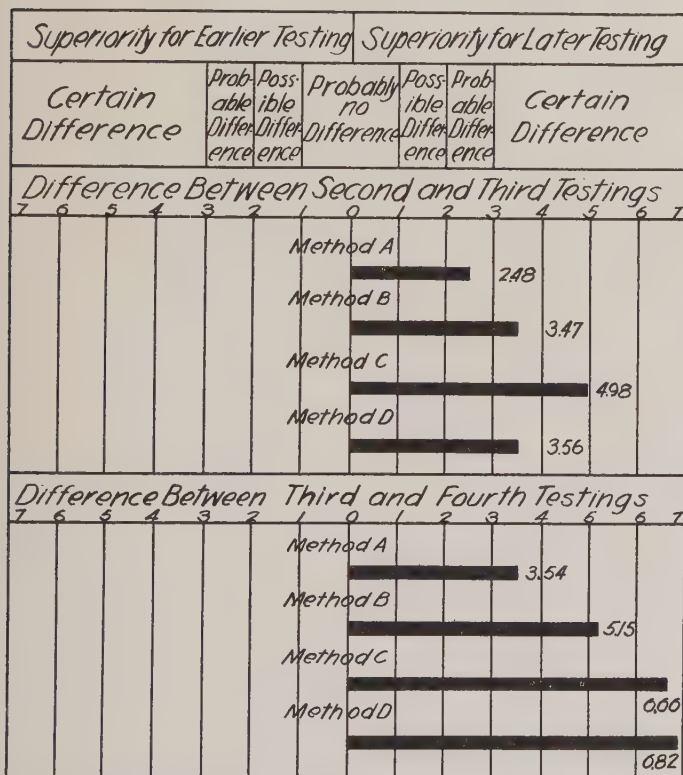


FIG. 13.—The significance of the differences between testings, expressed in terms of the standard deviations of the differences.

Method A; on the third testing, Method B was possibly slightly superior and Method D was about equal to Method A; and, on the fourth testing, Methods B and D

were both almost certainly superior to Method A. In short, Methods B and D show no decided and reliable superiority to Method A until the fourth testing. This is probably due to the fact that the largest difference between the methods occurred on Types 18 to 28, and it will be shown later that most of the transfer to these types did not occur until the fourth testing.

TABLE XXXVII.—COMPARISON OF THE AMOUNTS OF TRANSFER OBTAINED BY EACH OF THE OTHER METHODS OF INSTRUCTION TO THAT OBTAINED BY METHOD A, ON EACH TESTING

		B-A	C-A	D-A
Second testing	Difference of means.....	2.8	-12.4	- 1.8
	S. D. of difference.....	3.87	3.74	3.67
	Diff. ÷ S. D. diff.....	.72	3.32	.49
	Percentage of increase.....	7.7	-34.1	- 5.0
Third testing	Difference of means.....	7.0	- 7.4	1.7
	S. D. of difference.....	4.05	3.86	3.95
	Diff. ÷ S. D. diff.....	1.73	1.92	.43
	Percentage of increase.....	15.2	-16.1	3.7
Fourth testing	Difference of means.....	12.8	3.2	12.2
	S. D. of difference.....	3.48	3.56	3.36
	Diff. ÷ S. D. diff.....	3.67	.90	3.63
	Percentage of increase.....	21.5	5.4	20.5

Table XXXVII also shows that Method C was almost certainly inferior to Method A on the second testing, that it was still somewhat inferior on the third testing, and that it was possibly slightly superior on the fourth testing. It would seem, therefore, that the rationalization, as employed in Method C, was a hindrance rather than a help, at first, and retarded the transfer. This same retardation is found in Method D, where the rationalization was combined with the generalization, but to a lesser

degree. It should be noted, however, that the amount of transfer obtained by Methods C and D is increasing more rapidly than that obtained by Method B (Table

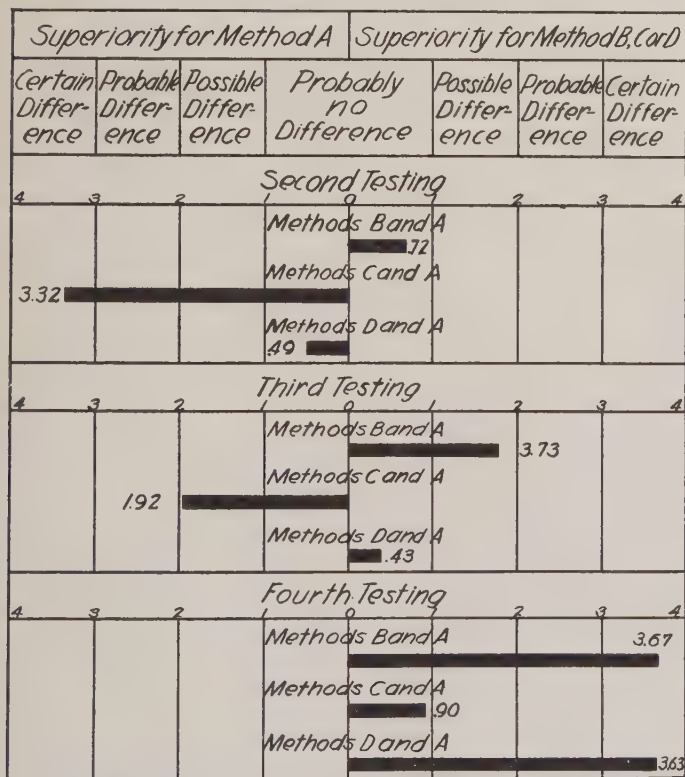


FIG. 14.—The significance of the difference between the different methods of teaching, for each testing, expressed in terms of the standard deviations of the differences.

XXXVI), which suggests that these methods might have passed Method B if the pupils had been tested a fifth time.

The reliability of the differences shown in Table XXXVII is presented graphically in Figure 14.

It has been shown (Table XXVIII) that the rationalization was relatively more effective with the pupils on the highest level of intelligence than for the others. This suggested that possibly the retarding effect of the rationalization, shown above, might be less marked with such pupils, or disappear entirely. In order to study this, Tables XXXVIII and XXXIX were compiled. Table XXXVIII shows the mean percentages of transfer, the

TABLE XXXVIII.—PERCENTAGE OF TRANSFER ON EACH TESTING FOR PUPILS ON THE HIGHEST LEVEL OF INTELLIGENCE

		Method of instruction			
		A	B	C	D
Second testing	Mean.....	38.2	38.4	27.1	35.9
	S. D.....	32.5	32.2	32.2	31.7
	S. D. of mean.....	5.28	5.23	5.23	5.15
Third testing	Mean.....	51.2	54.8	44.2	50.3
	S. D.....	31.5	37.6	29.0	35.2
	S. D. of mean.....	5.11	6.10	4.71	5.71
Fourth testing	Mean.....	65.0	84.3	75.7	85.0
	S. D.....	27.1	18.2	22.3	14.4
	S. D. of mean.....	4.40	2.99	3.62	2.34

standard deviations, and the standard deviations of the means, at the end of the second, third, and fourth testings, for pupils on the highest level of intelligence.

Table XXXIX was compiled from Table XXXVIII in order to make it easier to compare the methods of teaching. This table shows that the rationalization employed in Method C retarded the transfer almost as much for

pupils on the highest level of intelligence as it did for all of the pupils. Method C was probably inferior to

TABLE XXXIX.—COMPARISON OF EACH OF THE OTHER METHODS OF INSTRUCTION TO METHOD A, FOR EACH TESTING, AND FOR PUPILS ON THE HIGHEST LEVEL OF INTELLIGENCE

		B-A	C-A	D-A
Second testing	Difference of means.....	.2	-11.1	- 2.3
	S. D. of difference.....	7.43	7.43	7.38
	Difference of means S. D. of difference	.03	1.49	.31
	Percentage of increase over Method A.....	.5	-29.1	- 6.0
Third testing	Difference of means.....	3.6	- 7.0	- 0.9
	S. D. of difference.....	7.96	6.95	7.66
	Difference of means S. D. of difference	.45	1.01	.12
	Percentage of increase over Method A.....	7.0	-13.7	- 1.8
Fourth testing	Difference of means.....	19.3	10.7	20.0
	S. D. of difference.....	5.31	5.70	4.98
	Difference of means S. D. of difference	3.61	1.88	4.02
	Percentage of increase over Method A.....	29.7	16.5	30.8

Method A on the second and third testings, but was probably slightly superior on the fourth.

The reliability of the differences in Table XXXIX is less than for these in Table XXXVII because of the

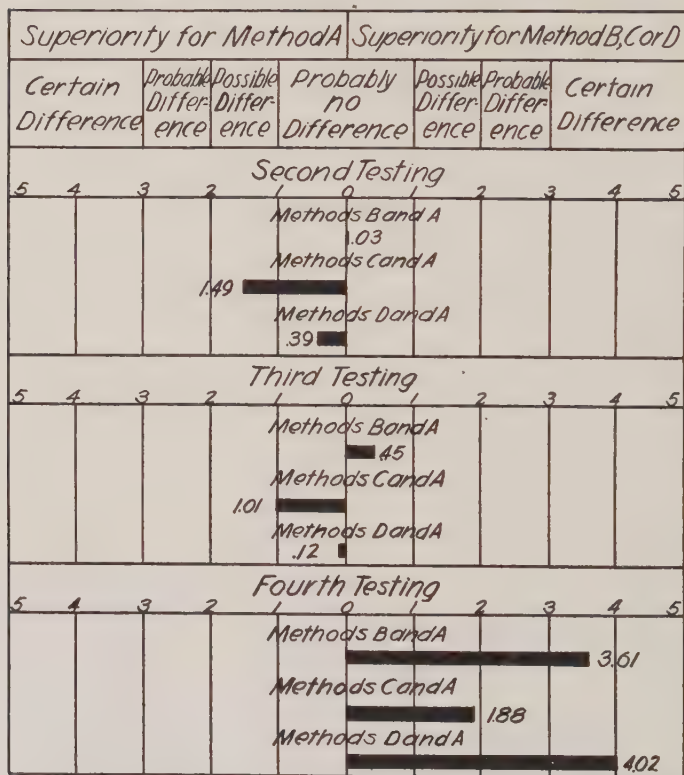


FIG. 15.—The significance of the differences between the different methods of teaching, on each testing, for pupils on the highest level of intelligence, expressed in terms of the standard deviations of the differences.

smaller number of pupils involved. The significance of the differences in Table XXXIX is shown graphically in Figure 15.

The retardation in transfer shown for Method C as compared to Method A, both for the entire group of pupils and for those on the highest level of intelligence, was probably due to two causes. In the first place, Method C was slightly less effective than Method A in producing transfer to the examples of Types 7 to 17 and more effective on Types 18 to 28. Furthermore, it will be shown later that the greatest amount of transfer to Types 10 to 17 came on the second testing, whereas the greatest transfer to Types 18 to 28 did not come until the fourth. This would naturally result in a larger percentage of transfer on the fourth testing for Method C and smaller percentages on the second and third. The same was true to a lesser extent of Method D. Another factor that may also account for part of this retardation was the fact that the rationalization employed in Methods C and D involved a knowledge of place value on the part of the pupils. Many of the pupils were taught place value for the first time just before the start of the experiment and it is possible that it was not thoroughly understood when the experiment began. The increase in relative effectiveness of the rationalization in producing transfer, as shown on successive testings, may, therefore, have been partly the result of an increasing understanding of place value.

3. *Percentage of Pupils Getting Given Type Correct for First Time on Each Testing.*—The question as to when the transfer took place was also studied in another way. Table XL shows the percentage of pupils getting at least one example of a given type correct for the first time on each testing.¹ To illustrate, the first row of the table should be read as follows: 38.6 per cent of the 448 pupils

¹ Table XL was compiled from the complete records of the individual pupils. These can not be given here for lack of space.

got at least one example of Type 10 correct on the first testing, 70.2 per cent of the *remainder* of the pupils got at least one correct on the second testing, 65.9 per cent of the remainder got at least one on the third, and 39.3 per cent of the remainder got at least one on the fourth. Altogether, 93.8 per cent of the pupils who did not get any correct on the first testing got at least one correct on the subsequent testings.

The table shows that Types 10 to 17, which involve addition and subtraction of two and three place numbers, but which do not involve the placing of numbers having different numbers of digits, received the greatest transfer on the second testing, or after the pupils had been taught how to add two numbers of two digits each. Types 18 to 28, which involve the placing of numbers having different numbers of digits, received the greatest transfer on the fourth testing, or after the pupils had been taught to add a two, a two and a one place number in the order stated. This appearance of the greatest transfer just after teaching the type nearest related is just what we would expect on the assumption that the increase is due to transfer from the type taught, but is difficult to account for on any other assumption.

It should be noted that the transfer on any given type did not come all at once. To illustrate, let us take Type 11, which is the addition of two numbers of three digits each. Of the pupils who did not get at least one of this type on the first testing, 78.0 per cent got at least one on the second testing, after being taught to add two numbers of two digits each. Of the remainder, 68.3 per cent got at least one on the third, and 57.9 per cent of those then remaining got at least one on the fourth testing, although nothing new was taught after the second testing that had any direct bearing on Type 11. Alto-

gether 97.1 per cent of those who did not get any of this type on the first testing got at least one on subsequent testings. This raises several interesting questions. By the end of the fourth testing from 84.4 per cent to

TABLE XL.—PERCENTAGE OF PUPILS GETTING AT LEAST ONE EXAMPLE OF A GIVEN TYPE CORRECT FOR THE FIRST TIME ON EACH TESTING

Type of example	Testing				
	1st	2nd	3rd	4th	2nd-4th
10	38.6	70.2	65.9	39.3	93.8
11	39.1	78.0	68.3	57.9	97.1
12	35.5	60.2	39.1	35.7	84.4
13	34.6	67.9	56.4	56.1	93.9
14	30.1	63.6	53.5	30.2	88.2
15	48.9	73.8	46.7	43.8	92.1
16	38.2	51.6	47.8	48.6	87.4
17	39.7	62.6	46.5	44.4	88.9
18	17.0	40.9	31.8	86.0	93.5
19	13.6	33.9	30.5	59.0	81.1
20	10.0	23.1	21.6	56.4	73.7
21	3.3	18.0	16.6	40.9	59.6
22	4.2	15.4	18.5	30.7	52.2
23	7.8	18.6	17.6	50.5	66.8
24	3.6	16.4	16.1	41.9	59.3
25	2.5	13.0	15.8	34.1	51.7
26	19.9	25.9	26.7	43.6	69.1
27	3.3	19.4	18.1	33.6	56.1
28	8.5	20.5	20.6	42.9	63.9

93.9 per cent of the pupils had gotten at least one example of each of Types 10 to 17 correct. Would these percentages have continued to increase if the pupils had been given a fifth and sixth testing, after further practice but without being taught anything new; and would they

all have ultimately approximated 100 per cent? Another question concerns Types 18 to 28, which did not receive the greatest transfer until the fourth testing. Would these types have shown greater transfer if the pupils had been tested a fifth and a sixth time, and would this transfer ultimately have approximated that on Types 10 to 17?

4. *Summary.*—The conclusions already drawn as to when the transfer occurred may be summarized as follows:

1. There was a steady increase in the amount of the transfer from the second to the fourth testings.

2. This increase was greatest for Method C, next greatest for Method D and least for Method A.

3. The rationalization was a handicap on the second and third testings and reduced the amount of transfer. On the fourth testing, however, Method C produced as much transfer as Method A and Method D produced as much as Method B.

4. The greatest transfer to any given type of example occurred on the next testing after the pupils had been taught the type most nearly related. This is strong evidence that the improvement on the untaught types was largely due to transfer from the instruction and practice on related types.

5. The transfer to a given type of example did not all occur on the testing immediately following the teaching of the nearest related type; but continued to occur, on successive testings, in diminishing amounts.

VII. ANALYSIS OF ERRORS

In order to discover the types of errors made by the pupils, and, if possible, to determine the sources of these errors, a careful study was made of the incorrect answers

to four typical examples. The first example of Type 11, the addition of two three-place numbers; the second example of Type 17, the subtraction of three-place numbers; Type 22, the addition of a two-place and a three-place number; and the second example of Type 27, the subtraction of a two-place from a three-place number, were chosen for this purpose.

1. *Analysis of Errors on Type 11.*—A summary of the errors made on the first example of Type 11 is given in Table XLI. This was the addition example 274. Table

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XLI shows that 79 of the 112 pupils taught by Method A attempted this example on the first testing. Of these 45.6 per cent worked it correctly and an additional 15.2 per cent used the correct method but

TABLE XLI.—ANALYSIS OF ERRORS ON FIRST EXAMPLE OF
TYPE 11

	First testing				Last testing			
	Method				Method			
	A	B	C	D	A	B	C	D
Number attempted.....	79	66	75	76	112	107	110	108
Percentage correct.....	45.6	60.6	57.3	51.3	91.1	95.3	92.7	95.4
Percentage making errors on combinations only.....	15.2	9.1	6.7	7.9	8.9	4.7	3.6	2.8
Percentage adding digits.....	19.0	15.2	16.0	13.2	0.0	0.0	0.0	0.9
Percentage of unclassified errors	20.3	15.2	20.0	27.6	0.0	0.0	3.6	0.9

made one or more errors on the combinations involved. The remaining 39.3 per cent of those who tried the example used incorrect methods. About half of them, or 19.0 per cent of the whole number attempting the example, added all of the digits of both numbers and obtained 19 as the result. The prevalence of this error was

probably due to negative transfer or interference. These pupils had never had any experience in adding numbers of two and three places, but had added columns

$$\begin{array}{r} 3 \\ \text{such as } 2, \text{ in which they had to add all of the num-} \\ \underline{1} \end{array}$$

bers to obtain the result. In the absence of any knowledge of the correct method they apparently applied the former method to the new situation. The remaining 20.3 per cent of those who tried the example got a variety of answers, no two of which were alike.

On the last testing all of the 112 pupils taught by Method A attempted this example, 91.1 per cent worked it correctly and the others all employed the correct method but made errors on the combinations. The incorrect method of adding the digits and the random, unclassified errors of the first testing were entirely missing. All of this certainly indicates that the instruction given on the addition of two numbers of two digits each transferred to the addition of two numbers of three digits each, without any substantial loss.

The above discussion has been confined entirely to the pupils taught by Method A. An examination of Table XLI shows that the pupils taught by the other methods made the same errors in substantially the same proportions.

2. *Analysis of Errors on Type 17.*—An analysis of the errors made on the second example of Type 17 is given in Table XLII. This was the subtraction example 687. Table XLII shows that 74 of the 112 pupils

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 taught by Method A attempted this example on the first testing. Of these, 52.7 per cent worked the

example correctly; an additional 5.4 per cent used the correct method but made errors on combinations; 2.7 per cent added instead of subtracting, obtaining 8118 as the result; 8.1 per cent added some of the digits and subtracted others, 858 being the most

TABLE XLII.—ANALYSIS OF ERRORS ON SECOND EXAMPLE OF TYPE 17

	First testing				Last testing			
	Method				Method			
	A	B	C	D	A	B	C	D
Number attempted.....	74	63	73	61	108	106	112	106
Percentage correct.....	52.7	73.0	50.7	55.7	87.0	93.4	78.6	90.6
Percentage making errors on combinations only.....	5.4	4.8	5.5	3.3	7.4	3.8	7.1	3.8
Percentage adding.....	2.7	3.2	5.5	1.6	0.9	0.9	0.0	0.0
Percentage adding and subtracting.....	8.1	4.8	5.5	6.6	0.9	0.9	2.7	0.9
Percentage adding digits.....	1.4	3.2	0.0	1.6	0.0	0.0	0.0	0.0
Percentage of unclassified errors	29.7	11.1	32.9	31.1	3.7	0.9	11.6	4.7

frequent answer; 1.4 per cent added all of the digits; and the remaining 29.7 per cent obtained a wide variety of results. As in the case of Type 11, all of the incorrect methods of procedure commonly used were apparently the result of attempting to apply to the new situation habits formed in the previous work.

On the last testing 108 of the 112 pupils taught by Method A attempted this example. Of these 87.0 per cent worked it correctly and an additional 7.4 per cent used the correct method. As in the case of Type 11, the incorrect methods and the unclassified errors were greatly reduced. These figures indicate that the effects of the instruction on two-place addition transferred to three-place subtraction.

A study of Table XLII shows that Methods B and D produced somewhat better results than Method A on the last testing, and that Method C produced somewhat poorer. The differences, however, are rather small in all cases. The comparatively high percentage of unclassified errors for Method C on the last testing may indicate that the rationalization tended to confuse some of the pupils.

3. *Analysis of Errors on Type 22.*—Table XLIII gives a summary of the errors made on Type 22. This was the addition example 24. Table XLIII shows that 83 of

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the 112 pupils taught by Method A attempted this example on the first testing. Of these 3.6 per cent worked it correctly; 1.2 per cent used the correct method

TABLE XLIII.—ANALYSIS OF ERRORS ON TYPE 22

	First testing				Last testing			
	Method				Method			
	A	B	C	D	A	B	C	D
Number attempted.....	83	82	83	91	104	103	108	112
Percentage correct.....	3.6	2.4	14.5	2.2	36.5	64.1	42.6	54.5
Percentage making errors on combinations only.....	1.2	0.0	0.0	1.1	4.8	1.0	3.7	1.8
Percentage keeping left column straight.....	4.8	18.3	14.5	11.0	7.7	10.7	9.3	11.6
Percentage keeping both left and right columns straight.....	2.4	3.7	1.2	3.3	1.0	3.9	13.9	7.1
Percentage adding same figure in two places.....	2.4	4.9	3.6	5.5	1.9	1.0	0.9	6.3
Percentage failing to bring down figure.....	4.8	12.2	8.4	4.4	7.7	2.9	8.3	5.4
Percentage adding extra figure to nearest column.....	4.8	3.7	9.6	2.2	5.8	3.9	4.6	0.9
Percentage adding digits.....	4.8	12.2	3.6	11.0	1.0	0.0	0.0	0.9
Percentage of unclassified errors	71.1	42.7	44.6	59.3	33.7	12.6	16.7	11.6

but made errors on the combinations; 4.8 per cent kept the left column straight, obtaining 562 as the result; 2.4 per cent wrote the 2 of the 24 in hundreds place and the 4 in units place, getting 526 as the answer; 2.4 per cent added one digit of the 24 to two digits of the 322, getting either 546 or 566 as the answer; 4.8 per cent failed to bring down the extra figure of the 322, obtaining 46 or 56 as the result; 4.8 per cent added the extra figure of the 322 to the next column, getting 76 or 58; 4.8 per cent added all of the digits and got 13 for the answer; and 71.1 per cent got a wide variety of answers. The pupils taught by the other methods made the same types of errors.

Obviously, the two greatest difficulties were the placing of the numbers and the question of what to do with the extra digit of the 322. The pupils had had no previous experience in handling either of these difficulties and apparently tried all possible ways. It is interesting to note that the plan of keeping the left column straight was considerably more popular than that of keeping the right column straight. For example, of the pupils taught by Method B who attempted this example, 2.4 per cent followed the former method and 18.3 per cent the latter. This was undoubtedly due to transfer of the habit of keeping the left margin straight in all of their written work.

On the final testing 104 of the 112 pupils taught by Method A attempted this example. Of these 36.5 per cent worked it correctly and another 4.8 per cent used the right method. The pupils taught by Method B showed much greater improvement than those taught by Method A. Of the 103 who attempted this example, 64.1 per cent worked it correctly and an additional 1.0 per cent employed the right method. Method D ranked

second and Method C third. The same types of errors occurred as on the first testing but usually less frequently, the greatest reduction being in the unclassified errors.

4. *Analysis of Errors on Type 27.*—A summary of the errors made on the second example of Type 27 is given in Table XLIV. This was the subtraction example 574. An examination of Table XLIV shows

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TABLE XLIV.—ANALYSIS OF ERRORS ON SECOND EXAMPLE OF TYPE 27

	First testing				Last testing			
	Method				Method			
	A	B	C	D	A	B	C	D
Number attempted.....	53	67	66	71	102	93	110	106
Percentage correct.....	1.9	3.0	6.1	5.6	39.2	65.6	44.5	55.7
Percentage making errors on combinations only.....	5.7	1.5	3.0	2.8	2.0	1.1	1.8	1.9
Percentage keeping left column straight.....	5.7	14.9	13.6	11.3	5.9	3.2	4.5	6.6
Percentage keeping both left and right columns straight.....	0.0	0.0	1.5	0.0	1.0	3.2	0.9	2.8
Percentage subtracting same figure in two places.....	1.9	6.0	4.5	4.2	4.9	0.0	4.5	0.9
Percentage failing to bring down figure.....	9.4	23.9	6.1	9.9	19.6	8.6	12.7	8.5
Percentage adding instead of subtracting.....	3.8	1.5	6.1	11.3	2.9	2.2	7.3	2.8
Percentage adding digits.....	3.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Percentage of unclassified errors	67.9	49.3	59.1	54.9	24.5	16.1	23.6	20.8

that the pupils had the same difficulties and made the same types of errors as on Type 22. Again the plan of keeping the left column straight was more popular on the first testing than that of keeping the right column straight; of the pupils taught by Method B, 14.9 per cent followed the first plan and 4.5 per cent the second.

As in the case of Type 22, Method B showed the greatest improvement on the last testing, Method D was second and Method C was third.

5. *Summary.*—The more important conclusions drawn from the above analysis of errors may be summarized as follows:

1. The errors in method that were made by any considerable number of pupils were usually the result of an attempt to apply to a new situation methods learned in connection with a different situation. In other words, these common errors of method were usually due to negative transfer or interference.

2. The analysis of errors strengthens the evidence previously given of transfer from the specific types taught in this experiment to the related types that were not taught. It shows that in each case a larger number of pupils attempted the example on the last testing than on the first and that a larger percentage of those attempting it worked it correctly.

CHAPTER IV

GENERAL SUMMARY

I. SUMMARY OF RESULTS

The more important of the results given in the preceding chapter are summarized below for convenient reference.

1. The effect of the instruction and practice given on certain specific types of examples was not confined to those types but spread, or transferred, to related types. Some spread occurred in the cases of 99.1 per cent of all the pupils. The mean transfer, for the group taught by the most favorable method, was 72.4 per cent of complete transfer. Of this group, 15.2 per cent obtained 100 per cent transfer. On the different types of examples the transfer ranged from 42.1 per cent to 90.4 per cent, for all of the pupils; and from 50.6 per cent to 92.9 per cent, for the group taught by the most effective method. In interpreting these figures the reader should keep in mind the method that was employed for measuring the transfer.

2. Some of the types of examples given on the test (Types 7, 8, 10 . . . 17) seemed to be so closely related to the specific types that were taught that the transfer took place automatically without any conscious generalization on the part of the pupils, or else the generalization was so simple that the majority of the pupils were able to make it without assistance. At any rate, there was little or no difference in the effectiveness of the different

methods of teaching in producing transfer to examples of these types.

3. On the types of examples where the generalization and rationalization would naturally be expected to be most effective, namely on those involving the placing of numbers having different numbers of digits, there were fairly large and reliable differences between the methods. On examples of this type, the generalization alone (Method B) increased the transfer by 45.1 per cent, the rationalization alone increased it by 15.5 per cent and the generalization and rationalization combined increased it 36.9 per cent. It appears, therefore, that aid in generalizing the process was an effective method of increasing transfer with pupils in the second grade.

4. There were no large or reliable differences between the two sexes with respect to transfer, although the generalization, as employed in Methods B and D, may possibly have been slightly more effective with the boys than with the girls.

5. In general there was an increase in the amount of transfer with an increase in mental age. This was greatest in the case of Method D (generalization and rationalization), somewhat less and about equal for Method B (generalization) and Method C (rationalization), and much less for Method A (neither generalization nor rationalization).

6. The generalization and rationalization were most effective in producing transfer with pupils in the highest third of the group with respect to mental age. No certain differences were found between the methods with pupils in the two lower thirds, although the evidence indicates that Method B was probably somewhat superior. With pupils in the highest third the generalization increased the transfer 29.7 per cent for all

examples and 49.5 per cent for Types 18 to 28, the rationalization increased the transfer by 16.5 per cent for all examples and 27.4 per cent for Types 18 to 28, and the generalization and rationalization combined increased the transfer by 30.8 per cent for all examples and by 56.2 per cent for Types 18 to 28. The fact that the generalization and rationalization combined increased the transfer for pupils in the highest third slightly more than the generalization alone, suggests that the combined generalization and rationalization might possibly be the most effective means of increasing transfer with older pupils.

7. The relative amounts of transfer to the different types of examples was not haphazard but could be accounted for as a result of the intrinsic difficulty of the different types and their degree of similarity or dissimilarity to the specific types that were taught.

8. The greatest transfer to any given type of example occurred on the next testing after the pupils had been taught the type most nearly related. All of the transfer did not take place on this testing, however, but it continued to occur, on successive testings, in diminishing amounts. This makes it seem possible that even larger transfer than that reported in this study might have been found if the pupils had been tested a fifth time.

9. The rationalization reduced the amount of transfer on the second and third testings. On the fourth testing, however, Method C (rationalization) produced as much transfer as Method A (neither rationalization nor generalization), and Method D (both rationalization and generalization) produced as much as Method B (generalization). This raises the question as to whether Methods C and D might not have exceeded Methods A and B respectively if the pupils had been tested a fifth time.

The results of the present investigation confirm those of the experiments by Ruediger, Judd, Ruger, Haskell, Hamblin, Coxe, Johnson, Meredith, and Woodrow, which were summarized in Chapter I, and which showed that large and useful amounts of transfer can be obtained if the securing of the greatest possible "spread" is made a conscious aim of instruction, and if the methods of teaching are intelligently chosen and employed with that aim in view. The mere fact that some other experimenters failed to find transfer does not necessarily mean that no transfer is possible, but may have been due to the fact that the method of instruction was such as to *specialize* rather than *generalize* the things taught. As Judd says:

Formalism and lack of transfer turn out to be not characteristics of subjects of instruction, but rather products of the mode of instruction in these subjects.¹

Those who have opposed the doctrine of formal discipline by saying that the school subjects at the present time do not give a generalized training are undoubtedly criticizing not the human mind, but our methods of instruction. It is, indeed, possible to find courses in arithmetic and algebra which are so narrowing in their effect upon students' minds that it is doubtful whether they ought to be included in the course of study at all. So long as narrow-minded teachers are put in charge of these courses, and especially if teachers become imbued with the idea that their only business is to cultivate a narrow and limited function of mind, we shall have examples of these pedagogical failures which in the statistics show that training has not been transferred.²

II. PEDAGOGICAL SIGNIFICANCE

This experiment was undertaken because the problem investigated was believed to be of fundamental importance in the teaching of arithmetic. It remains, there-

¹ Judd, C. H., *Psychology of High School Subjects*, 413.

² Judd, C. H. *op. cit.*, 424.

fore, to point out what seem to the writer to be the most important pedagogical implications of the results.

1. In the first place the results indicate that, while transfer from one type of example to a related type may occur in large amounts, and may be complete in the case of some individuals, it is seldom complete for the group as a whole. This means that instruction and practice on the fundamentals of arithmetic must be based upon full analyses of the fundamental processes. These should be analyses of learning difficulties rather than mathematical analyses. In view of the fact that transfer is seldom complete, all of the *essential* facts and all of the *essential* steps in the processes should be taught. Considerable experimental work needs to be done to determine what constitute the essential facts and the essential steps in the learning of each of the processes.

2. Although transfer from one type of example in arithmetic to other related types is possibly never complete, the results of this experiment show that it occurs in *useful* amounts—in amounts that we can not afford to ignore in view of the large number of facts and steps in the fundamental processes that must be taught and mechanized within a comparatively short period of time. In the present experiment it was found that, after the pupils had been given instruction and practice on addition examples of the type 32, they were,

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without further instruction, 93.5 per cent accurate on addition examples of the type 232; 87.3 per cent

322

accurate on subtraction examples of the type 36;

24

and 84.4 per cent accurate on subtraction examples

of the type 435. This does not mean that we can
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give the pupils instruction and practice on two place addition and ignore three place addition, and two and three place subtraction, entirely; but it does show that the problem of teaching the pupils how to handle the new types is a comparatively easy one and need not take very much time. It also indicates that it is not necessary to give extensive practice on each of these types separately, but that most of the drill can be of the mixed type and include all of them.

3. The effectiveness of a given method of teaching in securing the *immediate* end sought is not the sole test of its worth. The four methods of teaching employed in the present experiment were about equally effective with respect to the specific types taught, but differed widely in the amount of spread to related types. Our methods of teaching the fundamentals of arithmetic should be of such a character as to secure maximum transfer to related types as well as the best mastery of the specific types taught. *According to the results of this experiment this means that, in teaching any given type, we should not only teach that type, but also help the pupils to use it as a basis for generalizing the process.* Thus, in teaching

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examples of the type 32, the pupils should form the

2

generalization that the right hand column must be kept straight; or, in learning how to multiply by a two place number, they should form the generalization that the first figure of the product must be written under the figure by which they are multiplying. The results of this experiment indicate that the amount of transfer

can be substantially increased by a consistent and persistent emphasis on such generalizations.

The tendency of most of the current textbooks in arithmetic is to avoid all generalizations. This is probably the result of our well founded distrust of teaching by rule, which has led us to the extreme of rejecting rules and generalizations entirely. Thus Knight writes "There is considerable doubt about the efficiency of rules and definitions as aids to genuine understanding and insight as well as skill."¹ Rules and definitions arbitrarily imposed upon the pupil may be harmful; but when they are formulated by the pupil himself, as the result of his own experience, they are the culmination of the thought process and are essential to the broad understanding and application of the knowledge gained.

III. PROBLEMS FOR FURTHER EXPERIMENTATION

It seems to the writer that considerable further experimentation is needed on the general problem of methods of facilitating the transfer of training within the fundamentals of arithmetic. The present experiment attempted to measure the effectiveness of two factors in increasing transfer, namely, generalization and rationalization. The results obtained would seem to indicate that transfer may be substantially increased, in certain cases, by helping the pupils generalize the process. The results with regard to the rationalization, however, were not so conclusive.

Just how are we to interpret the fact that the rationalization, as used in this experiment, did not materially increase the transfer? Obviously several interpretations are possible. It might mean, for

¹ *Twenty-ninth Yearbook of the National Society for the Study of Education, Part I*, 264.

example, that rationalization is of no assistance to transfer. The experiment of Judd on hitting objects under water, however, would indicate that this is not the case. The results of this experiment alone certainly do not warrant any such conclusion.

At least two other explanations are possible. In the first place, it may be that rationalization is effective in increasing transfer with older pupils but not with pupils of the age studied in this experiment. In the second place, it may be that the examples employed in this study were not of a type in which rationalization is of any great assistance. At least it is possible to think of other types where it would probably be more effective. For example, if a pupil were taught the short method of multiplying by 25 and then tested on the short method of multiplying by $12\frac{1}{2}$, $33\frac{1}{3}$ and 50, it seems reasonable to suppose that greater transfer would occur if the principles underlying the short method of multiplying by 25 were considered in teaching that method. It would be profitable to carry out further experiments along this line.

Another problem which it was not possible to study very extensively in the present experiment is the problem of interference or negative transfer. The child constantly generalizes, whether we encourage him to do so or not, and if left to his own resources often forms false generalizations or generalizes too broadly. Many errors in arithmetic are probably due to this cause. Knight cites a number of instances of this kind under the heading of "interference factors." One illustration he gives is as follows:

Every child has many experiences in multiplication of whole numbers, all of which contribute to a notion that multiplication means getting larger. The following multiplications, however, behave in curious ways. Forty-eight times 1 results in 48 . . .

Forty-eight times 0 results in 0 . . . Forty-eight times one-fourth results in twelve—a great reduction in magnitude.¹

It is true that many children, from their experience with multiplication with whole numbers, form the sweeping generalization that the result is always larger than the number multiplied, and this often causes serious bewilderment when they come to multiplication by zero and by a proper fraction. The difficulty arises from the fact that they have generalized too widely and are trying to make a single generalization suffice where they need three. It seems reasonable to suppose that this confusion would be greatly lessened if the teacher made it his business to see that the proper generalizations were formed as soon as the pupils have the necessary experience.

It also seems reasonable to suppose that negative transfer, or interference, might often be lessened by rationalization. In the first cases of carrying that the pupils meet, the figure carried is usually 1. It is not at all uncommon for pupils to generalize from this and to fall into the error of always carrying 1. The writer recently examined a test paper written by a fourth grade pupil that contained four examples in multiplication. Every time the opportunity to carry arose the pupil had carried 1, no matter what the correct figure might be. Certainly it would seem that the number of mistakes of this kind might be greatly reduced if the pupils were taught the principles underlying the process.

To summarize, two suggestions have been made for further investigation to supplement the results of the present experiment: (1) Further experimentation on the effect of rationalization on transfer. (2) An experimental study of the effects of generalization and rationalization in reducing interference or negative transfer.

¹ *Twenty-ninth Yearbook of the National Society for the Study of Education, Part I*, 155.

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APPENDIX I
TABLES

TABLE XLVI.—DISTRIBUTION OF THE INITIAL SCORES OF THE 448 PUPILS, GROUPED ACCORDING TO SEX AND METHOD OF INSTRUCTION

Initial score (examples)	Number of pupils														
	Girls					Boys					Girls and boys				
	Method of instruction					Method of instruction					Method of instruction				
	A	B	C	D	All	A	B	C	D	All	A	B	C	D	All
40-45	4	4	2	4	14	4	7	5	5	21	8	11	7	9	35
35-39	8	13	12	11	44	8	10	8	8	34	16	23	20	19	78
30-34	6	5	6	5	22	7	5	5	8	25	13	10	11	13	47
25-29	5	8	8	7	28	11	9	14	5	39	16	17	22	12	67
20-24	14	13	10	10	47	12	14	9	12	47	26	27	19	22	94
15-19	9	7	10	10	36	10	13	11	17	51	19	20	21	27	87
10-14	6	2	3	4	15	5	1	5	3	14	11	3	8	7	29
5-9	1	1	2	2	6	2	0	2	1	5	3	1	4	3	11
0-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total.....	53	53	53	53	212	59	59	59	59	236	112	112	112	112	448
Mean.....	25.6	27.9	26.7	26.7	26.7	25.7	28.0	25.9	25.7	26.3	25.7	27.9	26.3	26.2	26.5
Median.....	23.8	27.2	25.9	25.4	25.4	25.2	25.8	25.5	23.0	25.1	24.4	26.5	25.9	24.3	25.2
S. D.....	9.8	8.9	9.4	9.9	9.5	9.3	8.5	9.6	8.8	9.1	9.5	8.7	9.5	9.3	9.3
Range.....	37	38	35	36	38	35	30	39	33	39	38	38	39	36	39

TABLE XLVII.—DISTRIBUTION OF THE TEACHER'S ESTIMATES OF THE 448 PUPILS, GROUPED ACCORDING TO SEX AND METHOD OF INSTRUCTION

Teacher's estimate	Number of pupils														
	Girls					Boys					Girls and boys				
	Method of instruction					Method of instruction					Method of instruction				
	A	B	C	D	All	A	B	C	D	All	A	B	C	D	All
A	13	3	8	7	31	10	5	3	2	20	23	8	11	9	51
B	17	23	18	23	81	25	25	22	23	95	42	48	40	46	176
C	15	23	21	19	78	19	17	27	29	92	34	40	48	48	170
D	8	4	4	3	19	4	12	5	4	25	12	16	9	7	44
F	0	0	2	1	3	1	0	2	1	4	1	0	4	2	7
Total.....	53	53	53	53	212	59	59	59	59	236	112	112	112	112	448
Mean ^a	2.7	2.5	2.5	2.6	2.6	2.7	2.4	2.3	2.4	2.4	2.7	2.4	2.4	2.5	2.5
Median.....	3.2	3.0	3.0	3.2	3.1	3.2	3.0	2.8	2.9	3.0	3.2	3.0	2.9	3.0	3.0
S. D.....	10.1	7.2	9.6	8.6	8.9	9.0	9.0	8.3	7.3	8.5	9.5	8.2	9.0	8.0	8.8
Range.....	3 ^b	3	4 ^c	4	4	4	3	4	4	4	4	3	4	4	4

^a These means were found by assigning values of 4, 3, 2, 1, 0 to the letters A, B, C, D, F, respectively and averaging in the usual way. All the means represent grades between C and B.

^b Three steps, i.e., from A to D.

^c Four steps, i.e., from A to F.

TABLE XLVIII.—DISTRIBUTION OF THE CHRONOLOGICAL AGES OF THE 448 PUPILS, GROUPED ACCORDING TO SEX AND METHOD OF INSTRUCTION

Age (months)	Number of pupils													
	Girls							Boys						
	Method of instruction							Method of instruction						
	A	B	C	D	All	A	B	C	D	All	A	B	C	All
115-119	0	1	0	1	2	1	0	0	1	2	1	1	0	4
110-114	4	3	4	3	14	3	4	4	3	14	7	7	8	28
105-109	5	5	3	3	16	3	7	7	5	22	8	12	10	38
100-104	4	4	8	5	21	4	5	7	3	19	8	9	15	40
95-99	14	23	11	15	63	19	17	15	16	67	33	40	26	130
90-94	19	14	19	20	72	23	22	20	20	85	42	36	39	157
85-89	7	3	8	6	24	6	4	6	11	27	13	7	14	51
Total	53	53	53	53	212	59	59	59	59	236	112	112	112	448
Mean	96.3	97.5	96.2	95.9	96.3	96.2	96.9	97.1	95.6	96.5	96.2	97.2	96.7	96.4
Median	95.2	97.1	94.9	95.2	95.8	95.1	96.0	96.2	94.6	95.4	95.2	96.6	95.6	95.6
S. D.	6.9	6.3	7.1	6.9	6.8	6.4	6.7	6.7	6.8	6.7	6.6	6.5	6.9	6.7
Range	26	28	28	29	30	28	26	26	29	29	29	28	28	30

TABLE XLIX.—RELIABILITY OF THE TESTS. RECORD OF FIFTY-THREE PUPILS ON TWO SUCCESSIVE TESTINGS, WITHOUT INSTRUCTION OR PRACTICE

Number of pupil	Sex	Mental age (months)	Number of examples correct		Number of pupil	Sex	Mental age (months)	Number of examples correct	
			First testing	Second testing				First testing	Second testing
1	G	83	3	3	27	B	76	3	3
2	G	77	4	5	28	B	89	19	19
3	G	94	18	14	29	B	75	9	7
4	G	75	14	19	30	B	79	6	2
5	G	79	16	14	31	B	85	21	8
6	G	80	0	5	32	B	82	19	15
7	G	90	8	1	33	B	79	2	4
8	G	92	0	10	34	B	84	17	19
9	G	97	3	9	35	B	70	4	5
10	G	84	11	14	36	B	93	16	14
11	G	86	14	15	37	B	93	14	12
12	G	94	19	18	38	B	70	16	5
13	G	98	22	22	39	B	85	18	18
14	G	92	17	16	40	B	71	9	14
15	G	86	18	21	41	B	99	22	32
16	G	95	15	14	42	B	99	11	1
17	G	73	5	2	43	B	98	11	23
18	G	82	16	13	44	B	117	20	20
19	G	87	17	19	45	B	91	18	18
20	G	80	6	5	46	B	98	19	16
21	G	94	20	20	47	B	100	13	16
22	G	106	29	29	48	B	103	18	16
23	G	94	15	17	49	B	102	19	22
24	G	104	17	19	50	B	115	40	33
25	G	103	32	36	51	B	115	18	19
26	G	97	13	16	52	B	101	19	30
					53	B	98	19	21
							99	30	40
			Total.....		791		4799		812
$r = .85 \pm .03$			Mean.....		14.9		90.5		15.3

TABLE L.—DISTRIBUTION OF TRANSFER TO ALL TYPES ON SECOND TESTING 448 PUPILS GROUPED ACCORDING TO METHOD OF INSTRUCTION

Percentage of transfer	Method of instruction				
	A	B	C	D	All
90 to 100	5	9	1	4	19
80 to 89.9	6	2	4	3	15
70 to 79.9	6	3	2	2	13
60 to 69.9	6	14	2	5	27
50 to 59.9	13	12	14	25	64
40 to 49.9	20	20	9	14	63
30 to 39.9	8	10	14	8	40
20 to 29.9	10	13	13	15	51
10 to 19.9	13	8	8	9	38
0 to 9.9	18	11	33	22	84
-10 to -0.1	4	6	4	3	17
-20 to -10.1	3	3	5	1	12
Below -20	0	1	3	1	5
Total	112	112	112	112	448
Mean	36.4	39.2	24.0	34.6	33.6
Median	40.0	43.0	22.3	36.3	34.3
S. D.	28.6	29.4	27.5	26.3	28.6
S. D. mean	2.70	2.77	2.59	2.48	1.35
Range	116.0	133.3	158.0	121.4	162.5

TABLE LI.—DISTRIBUTION OF TRANSFER TO ALL TYPES ON THIRD TESTING 448 PUPILS GROUPED ACCORDING TO METHOD OF INSTRUCTION

Percentage of transfer	Method of instruction				
	A	B	C	D	All
90 to 100	13	18	7	11	49
80 to 89.9	8	9	4	10	31
70 to 79.9	5	14	5	8	32
60 to 69.9	11	17	8	12	38
50 to 59.9	12	14	10	17	53
40 to 49.9	16	14	19	8	57
30 to 39.9	14	9	17	11	51
20 to 29.9	11	9	15	10	45
10 to 19.9	12	5	8	13	38
0 to 9.9	5	9	13	10	37
-10 to -	3	1	4	1	9
-20 to -	1	2	1	1	5
Below -20	1	1	1	0	3
Total.....	112	112	112	112	448
Mean.....	46.1	53.1	38.7	47.8	46.4
Median.....	45.6	53.3	38.2	51.2	46.3
S. D.....	30.0	30.7	27.9	29.3	29.9
S. D. mean.....	2.83	2.90	2.63	2.76	1.41
Range.....	127.3	123.1	133.3	111.1	133.3

TABLE LII.—DISTRIBUTION OF TRANSFER TO ALL TYPES ON FOURTH TESTING 448 PUPILS GROUPED ACCORDING TO SEX AND METHOD OF INSTRUCTION

Percentage of transfer	Number of pupils														
	Girls				Boys				Girls and boys						
	Method of instruction				Method of instruction				Method of instruction						
	A	B	C	D	All	A	B	C	D	All	A	B	C	D	All
90 to 100	12	15	10	17	54	9	23	11	13	56	21	38	21	30	110
80 to 89.9	4	8	4	10	26	10	8	9	14	41	14	16	13	24	67
70 to 79.9	4	10	8	2	24	4	8	7	8	28	16	18	16	10	52
60 to 69.9	6	3	7	8	24	10	5	7	12	34	8	8	14	20	58
50 to 59.9	4	4	6	4	18	8	3	6	7	28	12	11	12	11	46
40 to 49.9	6	5	4	4	21	4	3	6	1	14	10	8	12	5	35
30 to 39.9	8	4	7	1	20	5	1	4	2	12	13	5	11	3	32
20 to 29.9	6	3	2	3	14	4	1	3	0	8	10	4	5	3	22
10 to 19.9	0	0	2	3	5	4	1	3	1	9	4	1	5	4	14
0 to 9.9	3	1	1	1	5	1	2	2	1	6	4	3	2	2	11
-10 to -0.1	0	0	1	0	1	0	0	0	0	0	0	0	1	0	1
Total.....	53	53	53	53	212	59	59	59	59	236	112	112	112	112	448
Mean.....	58.8	70.0	62.0	70.0	65.2	60.3	74.6	63.6	73.5	68.0	59.6	72.4	62.8	71.8	66.7
Median.....	58.8	76.5	63.6	80.5	69.2	63.5	81.9	67.9	76.9	72.5	61.9	78.9	65.7	78.0	71.0
S. D.....	28.2	24.4	25.9	26.6	26.8	26.0	25.5	26.8	20.3	25.5	27.0	25.1	26.4	23.2	26.2
S. D. mean.....	3.87	3.35	3.56	3.65	1.84	3.39	3.32	3.49	2.64	1.66	2.55	2.37	2.49	2.19	1.24
Range.....	100.0	95.2	103.2	93.7	103.2	96.9	100.0	93.7	90.6	100.0	100.0	100.0	103.2	93.7	103.2

TABLE LIII.—DISTRIBUTION OF TRANSFER TO TYPES 7 TO 17 ON FOURTH TESTING 448 PUPILS GROUPED
ACCORDING TO METHOD OF INSTRUCTION

Percentage of transfer	Method of instruction				
	A	B	C	D	All
90 to 100	41	40	33	48	162
80 to 89.9	21	24	19	23	87
70 to 79.9	7	12	11	12	42
60 to 69.9	8	17	12	4	31
50 to 59.9	15	9	14	7	45
40 to 49.9	1	3	3	4	11
30 to 39.9	1	4	3	5	13
20 to 29.9	5	1	4	3	13
10 to 19.9	3	3	3	3	12
0 to 9.9	10	9	10	3	32
Total.....	112	112	112	112	448
Mean.....	73.2	72.6	67.2	77.1	72.5
Median.....	82.9	83.3	76.4	86.5	82.9
S. D.....	30.1	30.7	27.1	30.8	28.9
S. D. mean.....	2.85	2.90	2.56	2.91	1.86
Range.....	100.0	100.0	100.0	100.0	100.0

TABLE LIV.—DISTRIBUTION OF TRANSFER TO TYPES 18 TO 28 ON LAST TESTING 448 PUPILS GROUPED ACCORDING TO METHOD OF INSTRUCTION

Percentage of transfer	Method of instruction				
	A	B	C	D	All
90 to 100	28	47	31	35	141
80 to 89.9	5	8	5	15	33
70 to 79.9	6	10	11	17	34
60 to 69.9	2	7	6	11	26
50 to 59.9	10	11	10	9	40
40 to 49.9	4	0	7	4	15
30 to 39.9	11	5	7	6	29
20 to 29.9	9	8	6	7	30
10 to 19.9	14	8	9	8	39
0 to 9.9	23	8	19	10	60
-10 to -0.1	0	0	0	0	0
-20 to -10.1	0	0	1	0	1
Total.....	112	112	112	112	448
Mean.....	46.6	67.7	53.8	63.8	58.4
Median.....	39.1	79.0	57.0	71.4	63.8
S. D.....	35.6	33.4	35.6	32.8	35.3
S. D. Mean.....	3.37	3.16	3.37	3.09	1.67
Range.....	100.0	100.0	118.2	100.0	118.2

TABLE LV.—DISTRIBUTION OF TRANSFER ON THREE LEVELS OF INTELLIGENCE ON FOURTH TESTING. 448
PUPILS GROUPED ACCORDING TO METHOD OF INSTRUCTION

Percentage of transfer	Level I				Level II				Level III						
	Method of instruction				Method of instruction				Method of instruction						
	A	B	C	D	All	A	B	C	D	All	A	B	C	D	All
90 to 100	4	6	4	2	16	7	12	6	9	34	10	20	11	19	60
80 to 89.9	2	4	0	4	10	5	6	5	10	26	7	6	8	10	31
70 to 79.9	1	9	7	6	23	3	3	2	2	10	4	4	7	2	19
60 to 69.9	7	2	5	8	22	7	4	6	8	25	2	2	3	4	11
50 to 49.9	6	5	3	6	20	5	5	4	3	17	1	1	5	2	9
40 to 39.9	4	2	6	2	14	3	4	3	2	15	3	2	2	1	6
30 to 29.9	3	5	6	2	16	4	0	3	1	8	6	0	0	0	8
20 to 29.9	5	1	3	1	10	1	2	1	2	6	4	1	1	0	6
10 to 19.9	2	1	1	4	8	1	0	3	0	4	1	0	0	0	2
0 to 9.9	3	2	1	2	8	1	1	1	0	3	0	0	0	0	0
-10 to -0.1	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0
Total.....	37	37	37	37	148	37	37	37	37	148	38	38	38	38	152
Mean.....	49.8	62.6	54.4	57.2	56.0	63.8	70.1	58.0	73.0	66.2	65.0	84.3	75.7	85.0	77.5
Median.....	52.5	70.6	51.7	61.9	58.5	65.0	78.3	60.8	80.5	68.4	75.0	90.5	80.0	90.0	84.8
S. D.	26.1	26.3	25.9	25.3	26.3	25.2	25.0	25.9	20.7	24.9	27.1	18.2	22.3	14.4	22.6
S. D. mean.....	4.29	4.33	4.26	4.16	2.16	4.14	4.11	4.26	3.40	2.09	4.40	2.99	3.62	2.34	1.8
Range.....	96.9	100.0	103.2	93.7	103.2	100.0	100.0	92.3	71.9	100.0	86.7	71.4	80.0	58.6	86.74

APPENDIX II
TESTS

TEST No. 1

Name						Trial			
Add:									
4	5	2	6	5	7	4	7	8	6
2	3	2	1	2	1	3	2	1	2
—	—	—	—	—	—	—	—	—	—
		2	4					24	24
2	4	2	2			52	54	22	23
2	2	3	2	45	26	32	22	22	31
3	1	1	1	23	22	13	21	11	11
—	—	—	—	—	—	—	—	—	—
						242	242		
			526	464	222	232			
274	586	221	322	322	213	28	49		
213	212	132	211	111	211	14	25		
—	—	—	—	—	—	—	—	—	—

TEST No. 2

Name				Trial			
Subtract:							
6	8	4	7	7			
2	3	2	1	2			
—	—	—	—	—			
48	67	477	687				
23	22	212	231				

TEST No. 3
(To be dictated to pupils)

Add:						
	22	24	52	6	4	
74	22	23	2	32	32	
22	23	2	22	21	1	
—	—	—	—	—	—	—
					54	
542	357	24	235	6	322	
232	21	322	3	342	2	
—	—	—	—	—	—	—
Subtract:						
66	47	58	348	574	356	148
23	2	3	23	12	2	3
—	—	—	—	—	—	—

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TEST No. 3

Name _____

Trial _____

Add:

1	2	3	4	5	6
7	8	9	10	11	12

Subtract:

1	2	3	4	5	6	7
---	---	---	---	---	---	---

DIRECTIONS FOR ADMINISTERING THE TESTS

GENERAL DIRECTIONS

Have the pupils clear off their desks and see that each is supplied with a well sharpened *pencil*. Ask two or three pupils to distribute the tests placing them *face down* on the pupils' desks. When the tests are distributed and all the pupils are seated have them turn over the test papers and fill in the blanks at the top. The first time they take the test have them write *First* in the blank before *Trial*. The second time they take the test have them write *Second*, etc.

TEST NO. 1

This is not a speed test. Give the pupils all the time they can profitably use. When you are convinced that they have done all that they are able to do, take up the papers.

After the tests have been distributed and the blanks at the top properly filled, ask the pupils to hold up their pencils, resting their elbows on the desk, while you read the following directions.

Do not work any of the examples until I tell you to start. When I give the signal write the sums of as many of the examples as you can. If you do not know how to work an example do not write anything but go on to the next example. Start at the left of the first row and go across the paper, then work the second and third rows in the same order. (Show them.) Remember you are to *add*. Attention! Ready! Go!

TEST NO. 2

This is not a speed test. Give the pupils all the time they can profitably use. When you are convinced that they have done all that they are able to do, take up the papers.

After the tests have been distributed, and the blanks at the top properly filled, ask the pupils to hold up their pencils, resting their elbows on the desk, while you read the following directions:

Do not work any of the examples until I tell you to start. When I give the signal write the answers to as many of the examples as you can. If you do not know how to work an example do not write anything but go on to the next example. Start at the left of the first row and go across the paper, then work the second and the third rows in the same order. (Show them.) Remember you are to *subtract*. Attention! Ready! Go!

TEST NO. 3

This test is to be dictated to the pupils by the teacher. The work is to be done on the blanks provided. Always start dictating the numbers from the top. *Do not show them how to place the 22 under the 74, the 3 under the 45, etc.* The purpose of this test is to determine their ability to place the numbers in the proper place. After dictating the first number, allow the pupils sufficient time to get the answer before dictating the next example. The teacher can estimate the time necessary for writing the numbers and working the examples, by watching two or three of the pupils. When you come to the subtraction examples be sure the pupils understand that they are to *subtract*, not *add*.

After the papers have been distributed, and the blanks at the top properly filled, ask the pupils to hold up

their pencils, resting their elbows on the desk, while you read the following directions:

The first part of the test is *addition*. Write 74 at the top of the first square. (Hold up a blank and show them where.) Under the 74 write 22. *Add*. (Allow sufficient time, then continue.) At the top of the second square write 45. (Show them.) Under this write 3. *Add*. Etc, etc.

APPENDIX III
DIRECTIONS TO TEACHERS

DIRECTIONS TO TEACHERS

PRELIMINARY PREPARATION

Before starting the experiment the teacher should make sure that the pupils have mastered the following:

1. The ten addition combinations given at the beginning of Test No. 1. In thinking these combinations the pupils should *always start at the top*.

2. The five subtraction combinations given at the beginning of Test No. 2.

3. The addition of short columns of three and four numbers, sum less than ten, and involving only known combinations. as

$$\begin{array}{r} 5 \\ 3 \\ 1 \\ - \\ \hline \end{array} \quad \begin{array}{r} 4 \\ 2 \\ 2 \\ 1 \\ - \\ \hline \end{array}$$

In this work, as in the combinations, the pupils should *always start at the top*.

4. The reading and writing of numbers less than a thousand.

5. Such terms as addition, add, subtraction, subtract, sum, and column.

6. Place value to two places. The pupils should understand that 23 is 2 *tens* and 3 *ones*.

GROUP A

FIRST DAY

Teacher. We have been adding and subtracting numbers and today I want to find out how well you can do this work. First I am going to give you a test on addition. (Pass Test No. 1.) There are some examples on this test that are a little different from any you have had but I want to see if any of you can work them.

Give Test No. 1. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers and proceed as follows:

Teacher. Now I want to see how well you can subtract. (Pass Test No. 2.) There are some examples on this test that are a little different from any you have had but I want to see if any of you can work them.

Give Test No. 2. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers.

SECOND DAY

Teacher. Yesterday we had a test on addition and subtraction. Today I am going to give you a different kind of a test. (Pass the blanks for Test No. 3.) I am not sure that you can work these examples but I want to find out if any of you can.

Give test No. 3. Follow exactly the Directions for Administering.

THIRD DAY (20 MINUTES)

Teacher. On the test, yesterday and the day before, there were some addition examples that you did not know how to work because the numbers were larger than any you have ever added. Today I am going to show you how to add large numbers and some day I will give you the tests again.

Harry had 8 marbles. He bought 7 more. How do we find out how many marbles Harry now has? John, you may tell us.

John. Add 8 and 7.

Teacher. How many has he?

John. Fifteen.

Teacher. That is correct. Suppose Harry had 24 marbles and bought 22 more. How could we find out how many marbles Harry now has? Louise may tell us.

Louise. Add 24 and 22.

Teacher. That is correct, but we have never added such large numbers before, so I will show you how to do it. We want to add 24 and 22, so we write 24 (write 24 on board) and then write 22 under it. (Write 22 under 24.) Then we think, 4 and 2 are

$$\begin{array}{r} 24 \\ 22 \\ \hline 46 \end{array}$$

6, and write the 6 under the 2. Next we think 2 and 2 are 4, and write the 4 under the 2. Harry now has 46 marbles.

Tom and his sister Alice saved money for Mother's birthday present. Tom saved 44 cents, and Alice saved 32 cents. How many cents did they both save? We write the 32 under the 44. (Do this.) Then we think 4 and 2 are 6, and write the 6 under

$$\begin{array}{r} 44 \\ 32 \\ \hline 76 \end{array}$$

the 2. Next we think 4 and 3 are 7 and write the 7 under the 3. Tom and Alice saved 76 cents for their mother's birthday present.

Now we will see if you can add some large numbers. (Write the following on the board):

$$\begin{array}{r} 28 \\ 21 \\ \hline \end{array} \quad \begin{array}{r} 45 \\ 23 \\ \hline \end{array} \quad \begin{array}{r} 57 \\ 21 \\ \hline \end{array} \quad \begin{array}{r} 57 \\ 32 \\ \hline \end{array} \quad \begin{array}{r} 82 \\ 12 \\ \hline \end{array} \quad \begin{array}{r} 64 \\ 23 \\ \hline \end{array} \quad \begin{array}{r} 75 \\ 13 \\ \hline \end{array} \quad \begin{array}{r} 54 \\ 22 \\ \hline \end{array} \quad \begin{array}{r} 25 \\ 23 \\ \hline \end{array} \quad \begin{array}{r} 65 \\ 12 \\ \hline \end{array}$$

Mary you may go to the board and show us how to add the first example.

Mary. (Goes to board and points) 8 and 1 are 9. (Writes 9 under 1.) 2 and 2 are 4. (Writes 4 under 2.)

Teacher. 28 and 21 are how many?

Mary. 49.

Have the other examples worked in the same way. Then draw five large nuts on the board and write the following examples on them.

$$\begin{array}{r} 27 \\ 22 \\ \hline \end{array} \quad \begin{array}{r} 44 \\ 32 \\ \hline \end{array} \quad \begin{array}{r} 86 \\ 11 \\ \hline \end{array} \quad \begin{array}{r} 56 \\ 31 \\ \hline \end{array} \quad \begin{array}{r} 24 \\ 23 \\ \hline \end{array}$$

Teacher. Now we are going to play that we are Cracking Nuts.

To crack the nuts you must copy the examples and add. I want to see how many of you can crack all five. Stand when you have all of the nuts cracked.

Supervise this work closely. Be sure that all of the pupils understand the method of procedure. Help any who need assistance. Wait until about two-thirds of the pupils have finished and call for answers. Ask how many cracked all five of the nuts. How many cracked four?

If time permits write on the nuts other examples chosen from the Drill Examples given below. Handle in the same way.

Drill Examples

<u>45</u>	<u>78</u>	<u>26</u>	<u>47</u>	<u>52</u>	<u>58</u>	<u>57</u>	<u>45</u>	<u>66</u>
<u>23</u>	<u>21</u>	<u>21</u>	<u>32</u>	<u>32</u>	<u>31</u>	<u>21</u>	<u>22</u>	<u>12</u>
<u>65</u>	<u>44</u>	<u>56</u>	<u>86</u>	<u>25</u>	<u>64</u>	<u>47</u>	<u>26</u>	<u>57</u>
<u>12</u>	<u>23</u>	<u>32</u>	<u>12</u>	<u>22</u>	<u>13</u>	<u>21</u>	<u>22</u>	<u>32</u>
<u>74</u>	<u>67</u>	<u>42</u>	<u>54</u>	<u>76</u>	<u>56</u>	<u>78</u>	<u>27</u>	<u>48</u>
<u>13</u>	<u>11</u>	<u>22</u>	<u>23</u>	<u>12</u>	<u>31</u>	<u>11</u>	<u>21</u>	<u>31</u>
<u>55</u>	<u>76</u>	<u>24</u>	<u>46</u>	<u>47</u>	<u>57</u>	<u>46</u>	<u>67</u>	<u>56</u>
<u>32</u>	<u>22</u>	<u>23</u>	<u>21</u>	<u>22</u>	<u>22</u>	<u>22</u>	<u>12</u>	<u>22</u>
<u>58</u>	<u>54</u>	<u>48</u>	<u>77</u>	<u>68</u>	<u>27</u>	<u>46</u>	<u>28</u>	<u>57</u>
<u>21</u>	<u>33</u>	<u>21</u>	<u>12</u>	<u>11</u>	<u>22</u>	<u>32</u>	<u>21</u>	<u>31</u>
<u>54</u>	<u>87</u>	<u>62</u>	<u>74</u>	<u>25</u>	<u>85</u>	<u>75</u>	<u>54</u>	<u>66</u>
<u>32</u>	<u>12</u>	<u>12</u>	<u>23</u>	<u>23</u>	<u>13</u>	<u>12</u>	<u>22</u>	<u>21</u>
<u>56</u>	<u>44</u>	<u>65</u>	<u>68</u>	<u>52</u>	<u>46</u>	<u>74</u>	<u>62</u>	<u>75</u>
<u>21</u>	<u>32</u>	<u>23</u>	<u>21</u>	<u>22</u>	<u>31</u>	<u>12</u>	<u>22</u>	<u>23</u>
<u>47</u>	<u>76</u>	<u>24</u>	<u>45</u>	<u>67</u>	<u>56</u>	<u>87</u>	<u>72</u>	<u>48</u>
<u>32</u>	<u>11</u>	<u>22</u>	<u>32</u>	<u>21</u>	<u>22</u>	<u>11</u>	<u>12</u>	<u>31</u>
<u>55</u>	<u>67</u>	<u>44</u>	<u>64</u>	<u>74</u>	<u>75</u>	<u>64</u>	<u>76</u>	<u>65</u>
<u>32</u>	<u>22</u>	<u>23</u>	<u>12</u>	<u>22</u>	<u>22</u>	<u>22</u>	<u>21</u>	<u>22</u>
<u>85</u>	<u>45</u>	<u>84</u>	<u>77</u>	<u>86</u>	<u>72</u>	<u>64</u>	<u>82</u>	<u>75</u>
<u>12</u>	<u>33</u>	<u>12</u>	<u>21</u>	<u>11</u>	<u>22</u>	<u>23</u>	<u>12</u>	<u>13</u>

FOURTH DAY (20 MINUTES)

Teacher. I wonder if you remember what we learned yesterday about adding large numbers? (Write 46 on the board.) How

32

many of you know how to add these numbers? Herbert, you may show the class.

Herbert. (Goes to board and points) 6 and 2 are 8. (Writes 8 under 2.) 4 and 3 are 7. (Writes 7 under 3.)

Teacher. 46 and 32 are how many?

Herbert. 78.

Teacher. That is correct. Suppose we have 47 (Write 47 on board) and want to add 22 to it. Where should we write 22?

John, you may show the class.

John. (Goes to board and writes 22 under 47.)*

Teacher. Now you may show us how to add.

John. 7 and 2 are 9. (Writes the 9.) 4 and 2 are 6. (Writes the 6.)

Teacher. 47 and 22 are how many?

John. 69.

Teacher. That is correct. (Write on the board 45 56 26 65 57.) Harry you may go to the board and show us how to add 23 to 45.

Harry. (Goes to board and writes 23 under 45.) 5 and 3 are 8. (Writes 8.) 4 and 2 are 6. (Writes 6.)

Teacher. 45 and 23 are how many?

Pupils. 68.

Next have different pupils show how to add 31 to 56, 21 to 26, 12 to 65, and 21 to 57.

Then play Cracking Nuts. Use the following examples:

47	68	66	64	86
32	21	22	23	11

Teacher. I have put five nuts on the board. Copy and add. I want to see how many of you can crack all five. Some of you could not crack them all yesterday. Stand when you have finished.

Supervise this work carefully. Be sure that all the pupils understand the method of procedure. Help any who need assistance. Wait until about two-thirds of the pupils have finished and call for answers. Ask how many cracked all five? Four?

Continue as follows:

Teacher. Write 52 on your paper. Add 32 to it. What is your result, Henry?

Henry. 84.

Teacher. How many have 84?

*If the pupils do not know how to write the numbers, show them, without any explanation.

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Dictate the following in the same way:

62	74	48	65
<u>12</u>	<u>12</u>	<u>31</u>	<u>23</u>

Supervise this work carefully. If any of the pupils have difficulty in placing the numbers, or in adding, show them how.

Teacher. Harry has a new book that tells about trips in a flying machine. Last night he read 47 pages and today he read 31 pages. How many pages has he read? Raise your hand when you have the answer. How many pages has Harry read, Louise?

Louise. 78 pages.

Teacher. You may go to the board and show the class how you found the answer.

Louise. (Goes to board and writes 47.)

$$\begin{array}{r} 31 \\ 47 \\ \hline 78 \end{array}$$

Handle the following problems in the same way. Make up more, if necessary, using the numbers given in the Drill Examples. (Third Day.)

Mary earned 28¢ last week, helping her mother. This week she has earned 21¢. How much has she earned altogether?

Mr. and Mrs. Squirrel put away 54 nuts for winter. The squirrel children put away 22. How many nuts did they all put away?

Mr. and Mrs. Frog started a singing school. There were 27 froggies in Mrs. Frog's class and 22 in Mr. Frog's. How many froggies were in the singing school?

One morning Harry saw a large flock of wild geese flying north. He counted 65 geese. In the afternoon he saw a smaller flock and counted 13. How many geese did he count?

The first and second grades had a Flag Day drill. There were 44 first grade pupils and 32 second grade pupils. How many children were in the drill?

FIFTH DAY (20 MINUTES)

Teacher. Harry spent 52 cents for a reader and 32 cents for a speller. Find out how much he spent for both. Raise your hand when you have finished. How much did Harry spend?

John. 84 cents.

Teacher. You may show the class what you did.

John. (Goes to board and writes 52.)

$$\begin{array}{r} 32 \\ \underline{} \\ 84 \end{array}$$

Play Cracking Nuts. Use the following examples:

$$\begin{array}{r} 52 \\ \underline{} \end{array} \quad \begin{array}{r} 78 \\ \underline{} \end{array} \quad \begin{array}{r} 25 \\ \underline{} \end{array} \quad \begin{array}{r} 45 \\ \underline{} \end{array} \quad \begin{array}{r} 87 \\ \underline{} \end{array}$$

Teacher. I have put five nuts on the board. Copy and add. I want to see how many of you can crack all five. Some of you could not crack them all yesterday. Stand when you have finished.

Supervise this work closely. Be sure that all the pupils understand the method of procedure. Help any who need assistance. Wait until about two-thirds of the pupils have finished and call for answers. Ask how many cracked all five? Four?

Continue as follows:

Teacher. Write 47 on your paper. Add 32 to it. What is your result, Jane?

Jane. 79.

Teacher. How many have 79?

Dictate the following in the same way:

$$\begin{array}{r} 57 \\ \underline{} \end{array} \quad \begin{array}{r} 26 \\ \underline{} \end{array} \quad \begin{array}{r} 42 \\ \underline{} \end{array} \quad \begin{array}{r} 24 \\ \underline{} \end{array}$$

Supervise this work carefully.

Next divide the pupils into two equal groups and have a Relay Race. If there are an odd number of pupils appoint one as Judge. If there are an even number of pupils the teacher acts as Judge. Choose as many examples from the Drill Examples (Third Day) as there are pupils on each team. Write these on the board in two different places. Line the teams up, one in front of each group of examples. At a given signal the first pupil goes to the board and adds the first example. He then returns to his place in line and hands the chalk to the next pupil on his team. This pupil goes to the board and first makes any corrections he thinks necessary in the first example, then adds the next. Etc. Each pupil, before adding his example, may make corrections in any of the preceding

examples. The team that finishes first and has all of its examples right is the winner. If the team that finishes first does not have all of the examples correct it is disqualified. If neither team has all correct it is "no race." The judge declares the winner.

If the pupils have never held such a race before, describe it to them carefully before starting. If the teacher prefers and there are a suitable number of pupils, they may be divided into three teams. In any case the different teams should all be given the same examples.

If time permits a second race may be held.

SIXTH DAY (20 MINUTES)

Teacher. How many of you remember the test I gave you several days ago? Most of you did not work very many of the examples because you did not know how to add and subtract large numbers. We have been practicing on adding large numbers for several days so I am going to give you the same test again and I am sure you can work more examples this time. (Pass Test No. 1.) There are still some examples on the test that are a little different from any we have had but I believe some of you can work them.

Give Test No. 1. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers and proceed as follows:

Teacher. Now-I want to see how well you can subtract. We have not been practicing on subtraction for several days but what we have learned about adding large numbers may help you.

Give Test No. 2. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers.

If any time remains, give problems similar to the one given below. Use the numbers given in the Drill Examples (Third Day).

John and his brother Harry sell papers after school. Yesterday John sold 28 and Harry sold 21. How many did they both sell?

SEVENTH DAY (20 MINUTES)

Teacher. Yesterday we had a test on addition and subtraction.

Today I am going to give you a different kind of a test. (Pass blanks for Test No. 3.)

Give Test No. 3. Follow exactly the Directions for Administering. Use any time that remains for applied problems similar to those suggested for the Sixth Day.

EIGHTH DAY (20 MINUTES)

Teacher. Alice went to the store for her mother and bought a pound of butter for 45 cents and a half dozen eggs for 22 cents. Find how much she had to pay the clerk. What is your answer, John?

John. 67 cents.

Teacher. You may go to the board and show the class how you got it.

John. (Goes to board and writes 45.)

$$\begin{array}{r} 22 \\ 45 \\ \hline 67 \end{array}$$

Teacher. Harry went to the store for his mother and bought a cake of soap for 5 cents, a spool of thread for 3 cents, and a stick of chewing gum for himself for 1 cent. Find out how much he had to pay the clerk. What is your answer, Tom?

Tom. 9 cents.

Teacher. That is correct. You may go to the board and show the class how you got it.

Tom. (Goes to the board and writes 5.)

$$\begin{array}{r} 3 \\ 5 \\ \hline 8 \\ 1 \\ \hline 9 \end{array}$$

Teacher. Mary bought a doll for 32 cents, a doll hat for 22 cents, and a doll dress for 25 cents. What must we do to find out how much she had to pay for all three?

Pupils. Add.

Teacher. That is correct. We have never added *three* large numbers before, so I will show you how. First we write the 22 under the 32 just as we would if there were only two numbers

$$\begin{array}{r} 32 \\ 22 \\ 25 \\ \hline 79 \end{array}$$

to add. (Do this.) Then we write the 25 under the 22. (Do this.) Then we think 2 and 2 are 4, 4 and 5 are 9, and write 9 under the 5. (Do this.) Next we think 3 and 2 are 5, 5 and 2 are 7, and write 7 under the 2. How much did Mary pay for her doll clothes, Agnes?

Agnes. 79 cents.

Teacher. John stood at the window and counted the people who passed. He counted 55 men, 23 women, and 21 children. What must we do to find out how many people passed?

Pupils. Add.

Teacher. That is correct. We first write the 23 under the 55.

$$\begin{array}{r} 55 \\ 23 \\ 21 \\ \hline 99 \end{array}$$

Then we write the 21 under the 23. Then we think 5 and 3 are 8, 8 and 1 are 9, and write the 9. (Do this.) Then we think 5 and 2 are 7, 7 and 2 are 9, and write 9. (Do this.) How many people did John count?

Pupils. 99.

Next play Cracking Nuts. Write the following examples on the nuts:

26	52	42	46	45
22	32	32	21	23
31	13	23	12	11

After finishing the game, proceed as follows:

Teacher. Take pencil and paper. Add 24, 22, and 21. (Dictate slowly so pupils can write the numbers in proper place. Show them how to write the numbers, if necessary.) Raise your hand when you are through. What is your result, Dick?

Dick. 67.

Teacher. How many have 67?

Dictate in the same way, other examples taken from those given below. Supervise this work closely. Be sure that all the pupils understand the method of procedure. Help any who need assistance.

Drill Examples

22	44	55	25	26	45	26	42	45
22	22	22	23	21	22	21	22	22
<u>23</u>	<u>12</u>	<u>12</u>	<u>21</u>	<u>32</u>	<u>11</u>	<u>22</u>	<u>13</u>	<u>21</u>

62	46	24	26	46	42	24	46	45
22	21	22	22	21	22	22	22	22
<u>13</u>	<u>22</u>	<u>21</u>	<u>31</u>	<u>11</u>	<u>23</u>	<u>22</u>	<u>21</u>	<u>12</u>

52	45	62	25	46	26	44	52	25
22	22	12	22	21	22	23	32	22
<u>13</u>	<u>22</u>	<u>13</u>	<u>21</u>	<u>12</u>	<u>21</u>	<u>12</u>	<u>13</u>	<u>22</u>

54	42	44	56	24	44	45	55	42
23	32	23	21	23	23	23	23	32
<u>12</u>	<u>13</u>	<u>21</u>	<u>11</u>	<u>21</u>	<u>22</u>	<u>11</u>	<u>11</u>	<u>23</u>

26	44	46	56	25	45	56	24	46
21	23	21	21	22	23	22	23	22
<u>21</u>	<u>11</u>	<u>21</u>	<u>12</u>	<u>32</u>	<u>21</u>	<u>11</u>	<u>22</u>	<u>11</u>

54	46	66	46	65	56	22	55	45
23	31	11	31	13	32	22	23	33
<u>21</u>	<u>21</u>	<u>12</u>	<u>12</u>	<u>21</u>	<u>11</u>	<u>23</u>	<u>21</u>	<u>21</u>

44	24	46	65	44	26	52	46	56
33	23	31	13	22	21	32	32	21
<u>12</u>	<u>31</u>	<u>22</u>	<u>11</u>	<u>21</u>	<u>31</u>	<u>12</u>	<u>21</u>	<u>21</u>

56	66	46	66	46	62	54	56	45
21	12	31	12	32	12	22	22	33
<u>22</u>	<u>11</u>	<u>11</u>	<u>21</u>	<u>11</u>	<u>23</u>	<u>11</u>	<u>21</u>	<u>11</u>

54	24	24	25	25	55	54	24	22
23	22	22	22	23	22	23	23	22
<u>11</u>	<u>31</u>	<u>32</u>	<u>31</u>	<u>31</u>	<u>21</u>	<u>22</u>	<u>32</u>	<u>32</u>

NINTH DAY (20 MINUTES)

Teacher. Harry counted his building blocks before he put them away last night. He found that he had 44 white blocks and 32 red blocks. How many blocks has he? Raise your hand when you have the answer. How many blocks does Harry have, Jane?

Jane. 76.

Teacher. You may go to the board and show the class how you found the answer.

Jane. (Goes to board and writes 44.)

$$\begin{array}{r} 32 \\ 44 \\ \hline 76 \end{array}$$

Handle the following problems in the same way.

John went to the circus. He paid 25 cents to get in, 23 cents for candy, and 21 cents for peanuts. How much did he spend?

Mary Jane went to the park to see the animals. She counted 24 monkeys in one cage and 22 in another. How many monkeys did she count?

Mary Jane rode on a pony. That cost 25 cents. Then she took a ride on the elephant too. That cost 23 cents. How much did her rides cost?

The second grade had a party in the woods. Their teacher made sandwiches for the party. There were 26 ham sandwiches, 22 cheese sandwiches, and 21 peanut butter sandwiches. How many sandwiches were there?

John's father took him to the store and bought him some new clothes. His suit cost 24 dollars, his overcoat cost 23 dollars and the rest of his clothes cost 21 dollars. How much did they all cost?

Harry and his sister Agnes want to go to the country next summer for several weeks. Their board will cost them 48 dollars. To go to the country and come back will cost 21 dollars. How much will it cost them altogether?

Betty's family took an automobile trip one day last summer. They drove 46 miles before they stopped to eat dinner. After dinner they drove 21 miles and then stopped to visit some friends. They then drove 12 miles more before reaching home. How far did they drive?

Tom carries papers after school. He carries papers on three streets. On the first street he delivers 55 papers, on the second

street he delivers 23 papers, and on the last street 11 papers. How many papers does Tom deliver?

After working as many of these problems, as time permits, have a Relay Race. Use some examples with two numbers to be added and some with three. Select the examples from the lists of Drill Examples given for the Third and Eighth Days.

TENTH DAY (20 MINUTES)

Teacher. We know more about adding large numbers than we did the last time we took the tests, so I am going to let you take them again today to see if you can work more of the examples correctly.

Give Test No. 1. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers and give Test No. 2. If any time remains give applied problems similar to those suggested for the Ninth Day.

ELEVENTH DAY (20 MINUTES)

Teacher. Yesterday we took the first two of the tests on addition and subtraction. Today I am going to let you take the third test.

Give Test No. 3. Follow exactly the Directions for Administering. Use any time that remains for applied problems similar to those suggested for the Ninth Day.

TWELFTH DAY (20 MINUTES)

Teacher. Today I am going to show you something new about adding. We have been adding three numbers such as 52, 22 and 23. Esther, you may go to the board and show us how to add these numbers.

Esther. (Goes to board and writes 52.)

$$\begin{array}{r} 22 \\ 23 \\ \hline 97 \end{array}$$

Teacher. Alice went to the store for her mother and bought a pound of butter, a dozen eggs, and a box of matches. The butter cost 52 cents, the eggs 32 cents and the matches 3 cents. How would you find out how much Alice had to pay the clerk?

Pupils. Add.

Teacher. That is correct. The numbers we have been adding have all had two figures, but one of these numbers has only one figure, so I will show you how to write the numbers. We write

$$\begin{array}{r} 52 \\ 32 \\ \underline{3} \end{array}$$

the 32 under the 52, just the same as before. (Do this.) Now notice carefully where I write the 3. (Write 3 in proper place.) Now we are ready to add. We think 2 and 2 are 4, 4 and 3 are 7, and write the 7. (Do this.) Next we think 5 and 3 are 8, and write the 8. (Do this.) How much did Mary have to pay the clerk?

Pupils. 87 cents.

Teacher. We will work another problem to make sure we all understand.

Henry counted the number of steps he took in going from his house to the grocery store. He took 45 steps from his house to the corner, 22 from the corner to the door of the grocery store, and 2 from the door to the counter. How can we find out how many steps he took altogether?

Pupils. Add.

Teacher. We write the 22 under the 45, just as we have been doing.

$$\begin{array}{r} 45 \\ 22 \\ \underline{2} \end{array}$$

Now notice carefully where I write the 2. Now we are ready to add. We think 5 and 2 are 7, 7 and 2 are 9, and write the 9. (Do this.) Next we think 4 and 2 are 6 and write the 6. (Do this.) Henry took how many steps in going from his house to the grocery store?

Pupils. 69 steps.

Teacher. Suppose we want to add 44, 21 and 2. First we write the 44. (Do this.) You may show us where to write the 21, Mary.

Mary. (Goes to board and writes 21 below the 44.)

Teacher. You may show us where to write the 2, James.

James. (Goes to board and writes 2 in proper place.)

Teacher. You may go to the board and show us how to add, Betty.

Betty. (Goes to board and points.) 4 and 1 are 5, 5 and 2 are 7.

(Writes 7.) 4 and 2 are 6. (Writes 6.)

Teacher. Take pencil and paper. Write 62. Write 12 under 62.

Write 3 under 12. Add. What is your answer, Louise?

Louise. 77.

Teacher. How many have 77? 77 is correct. Now write 46.

Write 32 under the 46. Write 1 under the 32. Add. What is your answer, Harry?

Harry. 79.

Dictate more examples in the same way. Choose the examples from those given below. Supervise this work closely to make certain all are placing the numbers correctly. Always dictate the numbers from the top down, the two two-place numbers first, and the one-place number last.

If time permits, play Cracking Nuts. Take examples from those given below:

Drill Examples

44	22	55	25	26	45	26	42	45
22	22	22	23	21	22	21	22	32
<u>2</u>	<u>3</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>1</u>

62	46	24	42	24	52	44	25	46
22	21	22	32	22	32	21	22	22
<u>3</u>	<u>2</u>	<u>1</u>	<u>3</u>	<u>2</u>	<u>3</u>	<u>2</u>	<u>2</u>	<u>1</u>

62	75	56	76	55	64	62	24	85
12	22	32	11	33	22	32	23	12
<u>3</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>2</u>	<u>1</u>

72	44	52	54	74	86	25	82	56
12	32	22	32	23	11	22	12	21
<u>2</u>	<u>1</u>	<u>3</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>3</u>	<u>2</u>

THIRTEENTH DAY (20 MINUTES)

Have the following on the blackboard:

		45	52	46
78	85	22	32	21
<u>21</u>	<u>13</u>	<u>11</u>	<u>3</u>	<u>2</u>

25		62	42	86
22	47	12	32	11
32	22	3	23	2
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>

Teacher. There are ten addition examples on the board. Some of them are written correctly and some are not. Who sees one that is wrong? Which one do you think is wrong, Mary?

Mary. The second one.

Teacher. You may go to the board and correct it.

Mary. (Goes to the board and erases the 13 and writes it in proper place 85.)

$$\begin{array}{r} 13 \\ \hline \end{array}$$

Teacher. Is that right now?

Class. Yes.

Teacher. Who sees another one that is wrong?

Continue this until all have been corrected. Then have a Relay Race. Use a mixed list of examples similar to the ten given above, and chosen from the Drill Examples given for the Third, Eighth, and Twelfth days. Have some of the examples written with the addends incorrectly placed as in the ten examples given above. Each pupil is first to correct the writing of his example, if necessary, then add.

After the Relay Race proceed as follows:

Teacher. Last week John's mother gave him a new toy bank. John put 62 cents in his bank last week. He put 12 cents in yesterday, and 3 cents today. How many cents has he in the bank? Raise your hand when you have the answer. How many cents are in the bank, Mary?

Mary. 77 cents.

Teacher. You may go to the board and show the class how you found the answer.

Mary. (Goes to board and writes 62.)

$$\begin{array}{r} 12 \\ 3 \\ \hline 77 \end{array}$$

Handle the following problems in the same way. Make up more, if necessary, using the numbers given in the Drill Examples, for the Third, Eighth, and Twelfth Days.

Early in the morning a street car carried people to the city to work. Twenty-four got on where the car started. Twenty-three more got on at the next stop, and two more at the next stop. How many were on the car?

Mary's class had a school picnic. There were 26 girls and 21 boys. How many children were at the picnic?

Harry, James and Richard were playing marbles. Harry had 26 marbles, James had 21, and Richard had 31. How many marbles did they have?

Harold lives in the country. One Saturday last summer he set up a fruit stand by the road. He sold 24 baskets of apples, 22 baskets of peaches, and 32 baskets of grapes. How many baskets of fruit did he sell?

John and Agnes are twins. John weighs 42 pounds and Agnes weighs 32 pounds. How many pounds would they weigh standing on the scales together?

Louise went to the grocery for her mother and bought a can of peaches for 25 cents, a can of corn for 22 cents and some candy for herself for 1 cent. How many cents did she pay for all?

Harriet has read 56 pages in her new book. If she reads 32 pages today there will be only 11 pages left. How many pages are there in the book?

Mary and her mother were playing a number game. Mary's mother said "I am thinking of a number, can you guess what it is?" Mary guessed 52. "Oh, no!" her mother said. "Add 32 to that and you will have the right number." What number was Mary's mother thinking of?

Dorothy went to the toy store yesterday and bought some new clothes for her doll. She paid 42 cents for a dress, 22 cents for a pair of shoes and 3 cents for a hat. How much did the clothes cost?

FOURTEENTH DAY (20 MINUTES)

Teacher. We know more about adding large numbers than we did the last time we took the tests, so I am going to let you take them again today to see if you can work more of the examples correctly.

Give Test No. 1. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers and give Test No. 2. If any time remains give applied problems similar to those suggested for the Thirteenth Day.

FIFTEENTH DAY

Teacher. Yesterday we took the first two of the tests on addition and subtraction. Today I am going to let you take the third test.

Give Test No. 3. Follow exactly the Directions for Administering.

GROUP B

FIRST DAY

Teacher. We have been adding and subtracting numbers and today I want to find out how well you can do this work. First I am going to give you a test on addition. (Pass Test No. 1.) There are some examples on this test that are a little different from any you have had but I want to see if any of you can work them.

Give Test No. 1. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers and proceed as follows:

Teacher. Now I want to see how well you can subtract. (Pass Test No. 2.) There are some examples on this test that are a little different from any you have had but I want to see if any of you can work them.

Give Test No. 2. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers.

SECOND DAY

Teacher. Yesterday we had a test on addition and subtraction. Today I am going to give you a different kind of a test. (Pass the blanks for Test No. 3.) I am not sure that you can work these examples but I want to find out if any of you can.

Give Test No. 3. Follow exactly the Directions for Administering.

THIRD DAY (20 MINUTES)

Teacher. On the tests, yesterday and the day before, there were some addition examples that you did not know how to work because the numbers were larger than any you have ever added. Today I am going to show you how to add large numbers and some day I will give you the test again.

Harry had 8 marbles. He bought 7 more. How do we find out how many marbles Harry now has? John you may tell us.

John. Add 8 and 7.

Teacher. How many has he?

John. Fifteen.

Teacher. That is correct. Suppose Harry had 24 marbles and bought 22 more. How could we find out how many marbles Harry now has? Louise may tell us.

Louise. Add 24 to 22.

Teacher. That is correct, but we have never added such large numbers before, so I will show you how to do it. We want to add 24 and 22, so we write 24 (write 24 on board) and then write

$$\begin{array}{r} 24 \\ 22 \\ \hline 46 \end{array}$$

22 under it. (Write 22 under 24.) Then we start adding at the right. We think (Point) 4 and 2 are 6, and write the 6 under the 2. Then we add the next column. We think (Point) 2 and 2 are 4, and write the 4 under the 2. Harry now has 46 marbles.

Tom and his sister Alice saved money for Mother's birthday present. Tom saved 44 cents, and Alice saved 32 cents. How many cents did they both save? We write the 32 under the

$$\begin{array}{r} 44 \\ 32 \\ \hline 76 \end{array}$$

44. (Do this.) Then we start to add at the right. We think (Point) 4 and 2 are 6, and write the 6 under the 2. Then we add the next column. We think (Point) 4 and 3 are 7, and write the 7 under the 3. Tom and Alice saved 76 cents for their mother's birthday present.

Suppose we want to add 58 and 31. (Write 58 on the board.)

$$\begin{array}{r} 58 \\ 31 \\ \hline \end{array}$$

Who can tell me what to do first?

Helen. Add the right-hand column.*

Teacher. (Pointing) 8 and 1 are how many?

Pupils. 9.

* If the pupils do not make this suggestion, the teacher should do so.

Teacher. Where should I write the 9?

Pupils. Under the right-hand column.*

Teacher. What should I do next?

Pupils. Add the next column.*

Teacher. 5 and 3 are how many?

Pupils. 8.

Teacher. Where shall I write the 8?

Pupils. Under the left-hand column.†

Teacher. 58 and 31 are how many?

Pupils. 89.

Teacher. Let us see what we have learned about adding large numbers. What is the first thing to do, Henry?

Henry. Add the right-hand column.†

Teacher. Where do we write the sum of the right-hand column, John?

John. Under the right-hand column.†

Teacher. What do we do next, Elizabeth?

Elizabeth. Add the left-hand column.†

Teacher. Where do we place the sum of the left-hand column, Harry?

Harry. Under the left-hand column.†

<i>Teacher.</i> (Write	28	45	57	57	82	64
	<u>21</u>	<u>23</u>	<u>21</u>	<u>32</u>	<u>12</u>	<u>23</u>
	75	54	25	65		
	<u>13</u>	<u>22</u>	<u>23</u>	<u>12</u>	on the board.)	

Mary, you may go to the board and show us how to add the first example.

Mary. (Goes to board and points) 8 and 1 are 9. (Writes 9 under the right-hand column.) 2 and 2 are 4. (Writes 4 under the left-hand column.)

Teacher. 28 and 21 are how many?

Mary. 49.

Have as many of the other examples worked in the same way as time permits. Add more, if necessary, choosing them from the Drill Examples given below.

* If the pupils do not make this suggestion, the teacher should do so.

† If necessary, help the pupils formulate these generalizations.

Drill Examples

45	78	26	47	52	58	57	45	66
<u>23</u>	<u>21</u>	<u>21</u>	<u>32</u>	<u>32</u>	<u>31</u>	<u>21</u>	<u>22</u>	<u>12</u>

65	44	56	86	25	64	47	26	57
<u>12</u>	<u>23</u>	<u>32</u>	<u>12</u>	<u>22</u>	<u>13</u>	<u>21</u>	<u>22</u>	<u>32</u>

74	67	42	54	76	56	78	27	48
<u>13</u>	<u>11</u>	<u>22</u>	<u>23</u>	<u>12</u>	<u>31</u>	<u>11</u>	<u>21</u>	<u>31</u>

55	76	24	46	47	57	46	67	56
<u>32</u>	<u>22</u>	<u>23</u>	<u>21</u>	<u>22</u>	<u>22</u>	<u>22</u>	<u>12</u>	<u>22</u>

58	54	48	77	68	27	46	28	57
<u>21</u>	<u>33</u>	<u>21</u>	<u>12</u>	<u>11</u>	<u>22</u>	<u>32</u>	<u>21</u>	<u>31</u>

54	87	62	74	25	85	75	54	66
<u>32</u>	<u>12</u>	<u>12</u>	<u>23</u>	<u>23</u>	<u>13</u>	<u>12</u>	<u>22</u>	<u>21</u>

56	44	65	68	52	46	74	62	75
<u>21</u>	<u>32</u>	<u>23</u>	<u>21</u>	<u>22</u>	<u>31</u>	<u>12</u>	<u>22</u>	<u>23</u>

47	76	24	45	67	56	87	72	48
<u>32</u>	<u>11</u>	<u>22</u>	<u>32</u>	<u>21</u>	<u>22</u>	<u>11</u>	<u>12</u>	<u>31</u>

55	67	44	64	74	75	64	76	65
<u>32</u>	<u>22</u>	<u>23</u>	<u>12</u>	<u>22</u>	<u>22</u>	<u>22</u>	<u>21</u>	<u>22</u>

85	45	84	77	86	72	64	82	75
<u>12</u>	<u>33</u>	<u>12</u>	<u>21</u>	<u>11</u>	<u>22</u>	<u>23</u>	<u>12</u>	<u>13</u>

End the recitation as follows:

Teacher. Who will tell what we have learned today about adding large numbers? Henry, you may tell us.

Henry. First add the right-hand column, and write the sum under the right-hand column. Then add the left-hand column and write the sum under the left-hand column.

FOURTH DAY (20 MINUTES)

Teacher. Who remembers what we learned yesterday about adding large numbers? Herbert, you may tell the class.

Herbert. First add the right-hand column and write the sum under the right-hand column. Then add the left-hand column and write the sum under the left-hand column.

Teacher. That is correct. Suppose we want to add 47 and 21. (Write 47 on the board.) You may show me how to add, John.

$$\begin{array}{r} 21 \\ \hline \end{array}$$

John. (Goes to board and points) 7 and 1 are 8. (Writes 8 under the 1) 4 and 2 are 6. (Writes 6 under the 2.)

Teacher. 47 and 21 are how much?

John. 68.

Teacher. In adding large numbers we must be careful how we write the numbers. If we want to add 46 and 32, we must write them 46, not 46, or 46. An easy way to remember this

$$\begin{array}{r} 32 \\ \hline \end{array} \quad \begin{array}{r} 32 \\ \hline \end{array} \quad \begin{array}{r} 32 \\ \hline \end{array}$$

is to always watch the right-hand column and keep it straight like this 46 (trace down the right-hand column), not crooked

$$\begin{array}{r} 32 \\ \hline \end{array}$$

like this 46, or this 46. (Trace down the right-hand column.)

$$\begin{array}{r} 32 \\ \hline \end{array} \quad \begin{array}{r} 32 \\ \hline \end{array}$$

Suppose we have 65 (Writes 65 on board) and want to add 12 to it. Who can show me where to write the 12? Betty, you may show us.

Betty. (Goes to board and writes 65.)

$$\begin{array}{r} 12 \\ \hline \end{array}$$

Teacher. Who can tell the class everything we know about adding large numbers? Mary you may tell us.

Mary. First write the second number under the first with the right-hand column straight. Then add the right-hand column, and write the sum under the right-hand column. Then add the left-hand column, and write the sum under the left-hand column.

Teacher. That is correct. (Write on the board 45 56 26 65 57.) Harry you may go to the board and show us how to add 23 to 45.

Harry. (Goes to board and writes 23 under 45.) 5 and 3 are 8.

(Writes 8.) 4 and 2 are 6. (Writes 6.)

Teacher. 45 and 23 are how many?

Pupils. 68.

Next have different pupils show how to add 31 to 56, 21 to 26, 12 to 65, and 21 to 57. Then draw five large nuts on the board and write the following examples on them

47	68	66	64	86
<u>32</u>	<u>21</u>	<u>22</u>	<u>23</u>	<u>11</u>

Teacher. Now we are going to play that we are Cracking Nuts.

To crack the nuts you must copy the examples and add. I want to see how many of you can crack all five. Stand when you have all of the nuts cracked.

Supervise this work closely. Be sure that all of the pupils understand the method of procedure. Help any who need assistance. Wait until about two-thirds of the pupils have finished and call for answers. Ask how many cracked all five? How many cracked four?

Teacher. Write 52 on your paper. Add 32 to it. What is your result, Henry?

Henry. 84.

Teacher. How many have 84? Dictate the following in the same way:

62	74	48	65
<u>12</u>	<u>12</u>	<u>31</u>	<u>23</u>

Supervise this work carefully. If any of the pupils have difficulty in placing the numbers, or in adding, call on some other pupil to tell them how to add large numbers. (Right-hand column straight, etc.) Continue as follows:

Teacher. Harry has a new book that tells about trips in a flying machine. Last night he read 47 pages, and today he read 31 pages. How many pages has he read? Raise your hand when you have the answer. How many pages has Harry read, Louise?

Louise. 78 pages.

Teacher. You may go to the board and show the class how you found the answer.

Louise. (Goes to board and writes 47.)

$$\begin{array}{r} 31 \\ 47 \\ \hline 78 \end{array}$$

Handle the following problems in the same way. Make up more, if necessary, using the numbers given in the Drill Examples (Third Day).

Mary earned 28¢ last week, helping her mother. This week she has earned 21¢. How much has she earned altogether?

Mr. and Mrs. Squirrel put away 54 nuts for winter. The squirrel children put away 22. How many nuts did they all put away?

Mr. and Mrs. Frog started a singing school. There were 27 froggies in Mr. Frog's class and 22 in Mrs. Frog's. How many froggies were in the singing school?

One morning Harry saw a large flock of wild geese flying north. He counted 65 geese. In the afternoon he saw a smaller flock and counted 13. How many geese did he count?

The first and second grades had a Flag Day drill. There were 44 first grade pupils and 32 second grade pupils. How many children were in the drill?

End the recitation by having the pupils tell what they know about adding large numbers. Proceed as follows:

Teacher. How do we write large numbers when we want to add them, John?

John. We write the second number under the first with the right-hand column straight.

Teacher. What do we do next, Betty?

Betty. Add the right-hand column.

Teacher. Where do we write this sum?

Betty. Under the right-hand column.

Etc., etc.

FIFTH DAY (20 MINUTES)

Teacher. Who remembers what we learned yesterday about adding large numbers? Agnes you may tell the class.

Agnes. First write the second number under the first with the right-hand column straight. Then add the right-hand column, and write the sum under the right-hand column. Then add the left-hand column and write the sum under the left-hand column.

Teacher. Harry spent 52 cents for a reader and 32 cents for a speller. Find out how much he spent for both. Raise your hand when you have finished. How much did Harry spend?

John. 84 cents.

Teacher. You may show the class what you did.

John. (Goes to board and writes 52.)

$$\begin{array}{r} 32 \\ \underline{52} \\ 84 \end{array}$$

Play Cracking Nuts. Use the following examples:

$$\begin{array}{r} 52 \\ \underline{32} \end{array} \quad \begin{array}{r} 78 \\ \underline{21} \end{array} \quad \begin{array}{r} 25 \\ \underline{22} \end{array} \quad \begin{array}{r} 45 \\ \underline{22} \end{array} \quad \begin{array}{r} 87 \\ \underline{12} \end{array}$$

Teacher. I have put five nuts on the board. Copy and add. I want to see how many of you can crack all five. Some of you could not crack them all yesterday. Stand when you have finished.

Supervise this work closely. Be sure that all the pupils understand the method of procedure. Help any who need assistance. Wait until about two-thirds of the pupils have finished and call for answers. Ask how many cracked all five. Four? Continue as follows:

Teacher. Write 47 on your paper. Add 32 to it. What is your result, Jane?

Jane. 79.

Teacher. How many have 79?

Dictate the following in the same way:

$$\begin{array}{r} 57 \\ \underline{21} \end{array} \quad \begin{array}{r} 26 \\ \underline{22} \end{array} \quad \begin{array}{r} 42 \\ \underline{32} \end{array} \quad \begin{array}{r} 24 \\ \underline{23} \end{array}$$

Supervise this work carefully. If any of the pupils have difficulty in placing the numbers, or in adding, call on some pupil to tell them how to add large numbers. (Write the numbers with the right-hand column straight, etc.)

Next divide the pupils into two equal groups and have a Relay Race. If there are an odd number of pupils appoint one as Judge. If there are an even number of pupils the teacher acts as Judge. Choose as many examples from the Drill Examples (Third Day) as there are pupils on each team. Write these on the board in two different places. Line the teams up, one in front of each group of examples. At a given signal the first pupil goes to the board and adds the first example. He then returns to his place in line and hands the chalk to the next pupil on his team. This

pupil goes to the board and first makes any corrections he thinks necessary in the first example, then adds the next. Etc. Each pupil, before adding his example, may make corrections on any of the preceding examples. The team that finishes first and has all of its examples right is the winner. If the team that finishes first does not have all of the examples correct it is disqualified. If neither team has all correct it is "no race." The judge declares the winner.

If the pupils have never held such a race before, describe it to them carefully before starting. If the teacher prefers and there are a suitable number of pupils, they may be divided into three teams. In any case the different teams should all be given the same examples.

If time permits a second race may be held. End the recitation by having the pupils summarize what they know about adding large numbers.

SIXTH DAY (20 MINUTES)

Teacher. How many of you remember the test I gave you several days ago? Most of you did not work very many of the examples because you did not know how to add and subtract large numbers. We have been practicing on adding large numbers for several days so I am going to give you the same test again and I am sure you can work more examples this time. (Pass Test No. 1.) There are still some examples on the test that are a little different from any we have had but I believe some of you can work them.

Give Test No. 1. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers and proceed as follows:

Teacher. Now I want to see how well you can subtract. We have not been practicing on subtraction for several days, but what we have learned about adding large numbers may help you.

Give Test No. 2. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers.

If any time remains, give problems similar to the one given below. Use the numbers given in the Drill Examples (Third Day).

John and his brother Harry sell papers after school. Yesterday John sold 28 and Harry sold 21. How many did they both sell?

SEVENTH DAY (20 MINUTES)

Teacher. Yesterday we had a test on addition and subtraction.

Today I am going to give you a different kind of a test. (Pass blanks for Test No. 3.)

Give Test No. 3. Follow exactly the Directions for Administering. Use any time that remains for applied problems similar to those suggested for the sixth Day.

EIGHTH DAY (20 MINUTES)

Teacher. How many can tell us how to add large numbers?

Helen you may tell the class.

Helen. Write the second number under the first so the right-hand column is straight. Add the right-hand column and write the sum under the right-hand column. Add the left-hand column and write the sum under the left-hand column.

Teacher. Alice went to the store for her mother and bought a pound of butter for 45 cents and a half dozen eggs for 22 cents. Find out how much she had to pay the clerk? What is your answer, John?

John. 67 cents.

Teacher. You may go to the board and show the class how you got it.

John. (Goes to board and writes 45.)

$$\begin{array}{r} 22 \\ \underline{45} \\ 67 \end{array}$$

Teacher. Harry went to the store for his mother and bought a cake of soap for 5 cents, a spool of thread for 3 cents, and a stick of chewing gum for himself for 1 cent. Find out how much he had to pay the clerk. What is your answer, Tom?

Tom. 9 cents.

Teacher. That is correct. You may go to the board and show the class how you got it.

Tom. (Goes to the board and writes 5.)

$$\begin{array}{r} 3 \\ 5 \\ \underline{1} \\ 9 \end{array}$$

Teacher. Mary bought a doll for 32 cents, a doll hat for 22 cents, and a doll dress for 25 cents. What must we do to find out how much she had to pay for all three?

Pupils. Add.

Teacher. That is correct. We have never added *three* large numbers before, so I will show you how. First we write the 22 under the 32, keeping the right-hand column straight, just as we would if there were only two numbers to add. (Do

$$\begin{array}{r} 32 \\ 22 \\ 25 \\ \hline 79 \end{array}$$

this.) Then we write the 25 under the 22, still keeping the right-hand column straight. (Do this.) Then we think 2 and 2 are 4, 4 and 5 are 9, and write 9 under the 5. (Do this.) Next we think 3 and 2 are 5, 5 and 2 are 7, and write 7 under the 2. How much did Mary pay for her doll clothes, Agnes.

Agnes. 79 cents.

Teacher. John stood at the window and counted the people who passed. He counted 55 men, 23 women, and 21 children.

What must we do to find out how many people passed?

Pupils. Add.

Teacher. That is correct. We first write the 23 under the

$$\begin{array}{r} 55 \\ 23 \\ 21 \\ \hline 99 \end{array}$$

55, keeping the right-hand column straight. Then we write the 21 under the 23, still keeping the right-hand column straight. Then we think 5 and 3 are 8, 8 and 1 are 9, and write the 9. (Do this.) Then we think 5 and 2 are 7, 7 and 2 are 9, and write 9. (Do this.) How many people did John count?

Pupils. 99.

Teacher. How many can tell us how to add when we have three large numbers? Jack, you may tell us.

Jack. Write the second number under the first, and the third under the second, keeping the right-hand column straight. Then add the right-hand column, and write the sum under the

right-hand column. Then add the left-hand column and write the sum under the left-hand column.

Next play Cracking Nuts. Write the following examples in the nuts.

26	52	42	46	45
22	32	32	21	23
<u>21</u>	<u>13</u>	<u>23</u>	<u>12</u>	<u>11</u>

After finishing the game, proceed as follows:

Teacher. Take pencil and paper. Add 24, 22, and 21. (Dictate slowly so pupils can write the numbers in proper place. Show them how to write the numbers, if necessary.) Raise your hand when you are through. What is your result, Dick?

Dick. 67.

Teacher. How many have 67?

Dictate in the same way, other examples taken from those given below. Supervise this work closely. Be sure that all the pupils understand the method of procedure. Help any who need assistance.

Drill Examples

22	44	55	25	26	45	26	42	45
22	22	22	23	21	22	21	22	22
<u>23</u>	<u>12</u>	<u>12</u>	<u>21</u>	<u>32</u>	<u>11</u>	<u>22</u>	<u>13</u>	<u>21</u>
66	46	24	26	46	42	24	46	45
22	21	22	22	21	22	22	22	22
<u>13</u>	<u>22</u>	<u>21</u>	<u>31</u>	<u>11</u>	<u>23</u>	<u>22</u>	<u>21</u>	<u>12</u>
52	45	62	25	46	26	44	52	25
22	22	12	22	21	22	23	32	22
<u>13</u>	<u>22</u>	<u>13</u>	<u>21</u>	<u>12</u>	<u>21</u>	<u>12</u>	<u>13</u>	<u>22</u>
54	42	44	56	24	44	45	55	42
23	32	23	21	23	23	23	23	32
<u>12</u>	<u>13</u>	<u>21</u>	<u>11</u>	<u>21</u>	<u>22</u>	<u>11</u>	<u>11</u>	<u>23</u>
26	44	46	56	25	45	56	24	46
21	23	21	21	22	23	22	23	22
<u>21</u>	<u>11</u>	<u>21</u>	<u>12</u>	<u>32</u>	<u>21</u>	<u>11</u>	<u>22</u>	<u>11</u>

Drill Examples—(Continued).

54	46	66	46	65	56	22	55	45
23	31	11	31	13	32	22	23	33
<u>21</u>	<u>21</u>	<u>12</u>	<u>12</u>	<u>21</u>	<u>11</u>	<u>23</u>	<u>21</u>	<u>21</u>
44	24	46	65	44	26	52	46	56
33	23	31	13	22	21	32	32	21
<u>12</u>	<u>31</u>	<u>22</u>	<u>11</u>	<u>21</u>	<u>31</u>	<u>12</u>	<u>21</u>	<u>21</u>
56	66	46	66	46	62	54	56	45
21	12	31	12	32	12	22	22	33
<u>22</u>	<u>11</u>	<u>11</u>	<u>21</u>	<u>11</u>	<u>23</u>	<u>11</u>	<u>21</u>	<u>11</u>
54	24	24	25	25	55	54	24	22
23	22	22	22	23	22	23	23	22
<u>11</u>	<u>31</u>	<u>32</u>	<u>31</u>	<u>31</u>	<u>21</u>	<u>22</u>	<u>32</u>	<u>32</u>

End recitation by having pupils summarize what they know about adding *three* large numbers.

NINTH DAY (20 MINUTES)

Teacher. How many can tell us what we have learned about adding two large numbers? Joseph, you may tell us.

Joseph. Write the second under the first, keeping the right-hand column straight. Add the right-hand column, and write the sum under the right-hand column. Add the left-hand column, and write the sum under the left-hand column.

Teacher. That is fine. How many can tell us what we have learned about adding *three* large numbers? Hazel may tell us.

Hazel. Write the second under the first and the third under the second, keeping the right-hand column straight, etc.

Teacher. Harry counted his building blocks before he put them away last night. He found that he had 44 white blocks and 32 red blocks. How many blocks has he? Raise your hand when you have the answer. How many blocks does Harry have, Jane?

Jane. 76.

Teacher. You may go to the board and show the class how you found the answer.

Jane. (Goes to board and writes 44.)

$$\begin{array}{r} 32 \\ 76 \end{array}$$

Handle the following problems in the same way:

John went to the circus. He paid 25 cents to get in, 23 cents for candy, and 21 cents for peanuts. How much did he spend?

Mary Jane went to the park to see the animals. She counted 24 monkeys in one cage and 22 in another. How many monkeys did she count?

Mary Jane rode on a pony. That cost 25 cents. Then she took a ride on the elephant too. That cost 23 cents. How much did her rides cost?

The second grade had a party in the woods. Their teacher made sandwiches for the party. There were 26 ham sandwiches, 22 cheese sandwiches, and 21 peanut butter sandwiches. How many sandwiches were there?

John's father took him to the store and bought him some new clothes. His suit cost 24 dollars, his overcoat 23 dollars and the rest of his clothes cost 21 dollars. How much did they all cost?

Harry and his sister Agnes want to go to the country next summer for several weeks. Their board will cost them 48 dollars. To go to the country and come back will cost 21 dollars. How much will it cost them altogether?

Betty's family took an automobile trip one day last summer. They drove 46 miles before they stopped to eat dinner. After dinner they drove 21 miles and then stopped to visit some friends. They drove 12 miles more before reaching home. How far did they drive?

Tom carries papers after school. He carries papers on three streets. On the first street he delivers 55 papers, on the second street he delivers 23 papers, and on the last street 11 papers. How many papers does Tom deliver?

After working as many of these problems, as time permits, have a Relay Race. Use some examples with two numbers to be added and some with three. Select the examples from the lists of Drill Examples given for the Third and Eighth Days.

End the recitation by having the pupils summarize what they know about adding large numbers.

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TENTH DAY (20 MINUTES)

Teacher. We know more about adding large numbers than we did the last time we took the tests, so I am going to let you take them again today to see if you can work more of the examples correctly.

Give Test No. 1. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers and give Test No. 2. If any time remains give applied problems similar to those suggested for the Ninth Day.

ELEVENTH DAY (20 MINUTES)

Teacher. Yesterday we took the first two of the tests on addition and subtraction. Today I am going to let you take the third test.

Give Test No. 3. Follow exactly the Directions for Administering. Use any time that remains for applied problems similar to those suggested for the Ninth Day.

TWELFTH DAY (20 MINUTES)

Teacher. Today I am going to show you something new about adding. We have been adding three numbers such as 52, 22 and 23. Esther you may go to the board and show us how to add these numbers.

Esther. (Goes to board and writes 52.)

22

23

97

Teacher. Alice went to the store for her mother and bought a pound of butter, a dozen eggs, and a box of matches. The butter cost 52 cents, the eggs 32 cents and the matches 3 cents. How would you find out how much Alice had to pay the clerk?

Pupils. Add.

Teacher. That is correct. The numbers we have been adding have all had two figures, but one of these numbers has only one figure, so I will show you how to write the numbers. We

52

32

3

write the 32 under the 52, keeping the right-hand column straight, just the same as before. (Do this.) Now notice carefully where I write the 3. I must write it under the 2 so as to keep the right-hand column straight. (Write 3 in proper place.) Now we are ready to add. We think 2 and 2 are 4. 4 and 3 are 7, and write the 7. (Do this.) Next we think 5 and 3 are 8, and write the 8. (Do this.) How much did Mary have to pay the clerk?

Pupils. 87 cents.

Teacher. We will work another problem to make sure we all understand.

Henry counted the number of steps he took in going from his house to the grocery store. He took 45 steps from his house to the corner, 22 from the corner to the door of the grocery store, and 2 from the door to the counter. How can we find out how many steps he took altogether?

Pupils. Add.

Teacher. We write the 22 under the 45, keeping the right-hand column straight, just as we have been doing. Now notice carefully where I write the 2. I must write it under the 2 so as

$$\begin{array}{r} 45 \\ 22 \\ \underline{2} \end{array}$$

to keep the right-hand column straight. Now we are ready to add. We think 5 and 2 are 7, 7 and 2 are 9, and write the 9. (Do this.) Next we think 4 and 2 are 6 and write the 6. (Do this.) Henry took how many steps in going from his house to the grocery store?

Pupils. 69 steps.

Teacher. Suppose we want to add 44, 21 and 2. First we write the 44. (Do this.) You may show us where to write the 21, Mary.

Mary. (Goes to board and writes 21 below the 44.)

Teacher. You may show us where to write the 2, James.

James. (Goes to board and writes 2 in proper place.)

Teacher. You may go to the board and show us how to add, Betty.

Betty. (Goes to board and points) 4 and 1 are 5, 5 and 2 are 7. (Writes 7.) 4 and 2 are 6. (Writes 6.)

Teacher. Take pencil and paper. Write 62. Write 12 under 62. Write 3 under 12. Which column should be straight, Alice?

Alice. The right-hand column.

Teacher. All notice and see if you have the right-hand column straight. Add. What is your answer, Louise?

Louise. 77.

Teacher. How many have 77? 77 is correct. Now write 46. Write 32 under the 46. Write 1 under the 32. Be sure the right-hand column is straight. Add. What is your answer Harry?

Harry. 79.

Dictate more examples in the same way. Choose the examples from those given below. Supervise this work closely to make certain all are placing the numbers correctly. Constantly emphasize the fact that the numbers to be added must be written so that the right-hand column is straight. Always dictate the numbers from the top down, the two two-place numbers first, and the one-place number last.

If time permits, play Cracking Nuts. Take examples from those given below:

Drill Examples

44	22	55	25	26	45	26	42	45
22	22	22	23	21	22	21	22	32
<u>2</u>	<u>3</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>1</u>
62	46	24	42	24	52	44	25	46
22	21	22	32	22	32	21	22	22
<u>3</u>	<u>2</u>	<u>1</u>	<u>3</u>	<u>2</u>	<u>3</u>	<u>2</u>	<u>2</u>	<u>1</u>
62	75	56	76	55	64	62	24	85
12	22	32	11	33	22	32	23	12
<u>3</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>2</u>	<u>1</u>
72	44	52	54	74	86	25	82	56
12	32	22	32	23	11	22	12	21
<u>2</u>	<u>1</u>	<u>3</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>3</u>	<u>2</u>

THIRTEENTH DAY (20 MINUTES)

Have the following on the blackboard.

		45	52	46
78	85	22	32	21
<u>21</u>	<u>13</u>	<u>11</u>	<u>3</u>	<u>2</u>
25		62	42	86
22	47	12	32	11
<u>32</u>	<u>22</u>	<u>3</u>	<u>23</u>	<u>2</u>

Teacher. How many can tell us what we learned yesterday about writing numbers in addition. Jane you may tell the class.

Jane. You must write them so the right-hand column will be straight.

Teacher. There are ten addition examples on the board. Some of them are written correctly and some are not. Who sees one that is wrong? Which one do you think is wrong, Mary?

Mary. The second one.

Teacher. You may go to the board and correct it.

Mary. (Goes to the board and erases the 13 and writes it in proper place 85.)

13

Teacher. Is that right now?

Class. Yes.

Teacher. Who sees another one that is wrong?

Continue this until all have been corrected. Then have a Relay Race. Use a mixed list of examples similar to the ten given above, and chosen from the Drill Examples given for the Third, Eighth, and Twelfth Days. Have some of the examples written with the addends incorrectly placed as in the ten examples given above. Each pupil is first to correct the writing of his example, if necessary, then add.

After the Relay Race proceed as follows:

Teacher. Last week John's mother gave him a new toy bank.

John put 62 cents in his bank last week. He put 12 cents in yesterday, and 3 cents today. How many cents has he in the bank? Raise your hand when you have the answer. How many cents are in the bank, Mary?

Mary. 77 cents.

Teacher. You may go to the board and show the class how you found the answer.

Mary. (Goes to board and writes 62.)

$$\begin{array}{r} 12 \\ 3 \\ \hline 77 \end{array}$$

Handle the following problems in the same way. Make up more, if necessary, using the numbers given in the Drill Examples, for the Third, Eighth, and Twelfth Days.

Early in the morning a street car carried people to the city to work. Twenty-four got on where the car started. Twenty-three more got on at the next stop, and two more at the next stop. How many were on the car?

Mary's class had a school picnic. There were 26 girls and 21 boys. How many children were at the picnic?

Harry, James and Richard were playing marbles. Harry had 26 marbles, James had 21, and Richard had 31. How many marbles did they have?

Harold lives in the country. One Saturday last summer he set up a fruit stand by the road. He sold 24 baskets of apples, 22 baskets of peaches, and 32 baskets of grapes. How many baskets of fruit did he sell?

John and Agnes are twins. John weighs 42 pounds and Agnes weighs 32 pounds. How many pounds would they weigh standing on the scales together?

Louise went to the grocery for her mother and bought a can of peaches for 25 cents, a can of corn for 22 cents and some candy for herself for 1 cent. How many cents did she pay for all?

Harriet has read 56 pages in her new book. If she reads 32 pages today there will be only 11 pages left. How many pages are there in the book?

Mary and her mother were playing a number game. Mary's mother said "I am thinking of a number, can you guess what it is?" Mary guessed 52. "Oh, no!", her mother said, "Add 32 to that and you will have the right number." What number was Mary's mother thinking of?

Dorothy went to the toy store yesterday and bought some new clothes for her doll. She paid 42 cents for a dress, 22 cents for a pair of shoes and 3 cents for a hat. How much did the clothes cost?

End the recitation as follows:

Teacher. What must we remember in writing numbers in addition? Henry?

Henry. Keep the right-hand column straight.

Teacher. After we have the numbers written properly what do we do next, Florence?

Etc., etc.

FOURTEENTH DAY (20 MINUTES)

Teacher. We know more about adding large numbers than we did the last time we took the tests, so I am going to let you take them again today to see if you can work more of the examples correctly.

Give Test No. 1. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers and give Test No. 2. If any time remains give applied problems similar to those suggested for the Thirteenth Day.

FIFTEENTH DAY

Teacher. Yesterday we took the first two of the tests on addition and subtraction. Today I am going to let you take the third test.

Give Test No. 3. Follow exactly the Directions for Administering.

GROUP C

FIRST DAY

Teacher. We have been adding and subtracting numbers and today I want to find out how well you can do this work. First I am going to give you a test on addition. (Pass Test No. 1.) There are some examples on this test that are a little different from any you have had but I want to see if any of you can work them.

Give Test No. 1. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers and proceed as follows:

Teacher. Now I want to see how well you can subtract. (Pass Test No. 2.) There are some examples on this test that are a little different from any you have had but I want to see if any of you can work them.

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Give Test No. 2. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers.

SECOND DAY

Teacher. Yesterday we had a test on addition and subtraction.

Today I am going to give you a different kind of a test. (Pass the blanks for Test No. 3.) I am not sure that you can work these examples but I want to find out if any of you can.

Give Test No. 3. Follow exactly the Directions for Administering.

THIRD DAY (20 MINUTES)

Teacher. On the tests yesterday and the day before there were some addition examples that you did not know how to work because the numbers were larger than any you have ever added. Today we are going to find out how to add large numbers and some day I will give you the tests again.

Harry had 8 marbles. He bought 7 more. How do we find out how many marbles Harry now has? John you may tell us.

John. Add 8 and 7.

Teacher. How many has he?

John. Fifteen.

Teacher. That is correct. Suppose Harry had 24 marbles and bought 22 more. How could we find out how many marbles Harry now has? Louise may tell us.

Louise. Add 24 and 22.

Teacher. That is correct, but we have never added such large numbers before. Let us see if we can find out how to do it. (Writes 24 on the board.) How many of you remember about

22

ones and tens? That is fine. In adding large numbers we must break them up into ones and tens and add separately. How many tens are there in 24, Harry?

Harry. Two.

Teacher. How many ones are there?

Harry. Four.

Teacher. That is correct. So we can write "24 = 2 tens and 4 ones." (Writes as she talks.) Louise, how many tens are there in 22?

Louise. Two.

Teacher. How many ones are there?

Louise. Two.

Teacher. So what does 22 equal?

Pupils. 2 tens and 2 ones.

Teacher. (Writes "22 = 2 tens and 2 ones," in proper place.)

Now we are ready to add. 4 ones and 2 ones are how many ones, Agnes?

Agnes. 6 ones.

Teacher. (Writes "6 ones" in proper place.) 2 tens and 2 tens are how many tens, Arthur?

Arthur. 4 tens.

Teacher. (Writes "4 tens" in proper place.) 4 tens and 6 ones are how many?

Pupils. 46.

Teacher. (Writes 46 below 24.) Harry now has how many marbles?

22

Pupils. 46.*

Teacher. Elizabeth and Margaret sat in an automobile waiting for Mother. Elizabeth counted all the people who passed on one side of the street. There were 47. Margaret counted all the people who passed on the other side. There were 21. How would you find out how many people passed on both sides, John?

John. Add 47 and 21.

Teacher. (Writes 47 on the board.) What do we have to do to

21

big numbers so we can add them? Can you tell us Mary?

Mary. Break them up into tens and ones.†

Teacher. Harry what shall I write after the 47?

Harry. 4 tens and 7 ones.

Teacher. (Writes as Harry directs.) Ellen, what shall I write after the 21?

* The work on the board should now appear as shown below:

24 = 2 tens and 4 ones

22 = 2 tens and 2 ones

46 = 4 tens and 6 ones

† If the pupils do not make this suggestion, the teacher should do so.

Ellen. 2 tens and 1 one.

Teacher. (Writes as directed.) What should we do next? Can you tell us Edith?

Edith. See how many ones there are.*

Teacher. You may tell us how to find out, Edith.

Edith. 7 ones and 1 one are 8 ones.

Teacher. (Writes "8 ones" in proper place.) What must we do next?

Pupil. Find out how many tens there are.*

Teacher. Louis you may tell us how to find out.

Louis. 4 tens and 2 tens are 6 tens.

Teacher. (Writes 6 tens in proper place.) How many people passed the automobile?

Pupils. 68.

Teacher. (Writes 58 on the board.) I wonder if we could add
31

these numbers without writing so much on the board? Who can suggest what to do first?

Helen. Add the ones.*

Teacher. (Pointing) 8 and 1 are how many?

Pupils. 9.

Teacher. Where should I write the 9?

Pupils. Under the ones.*

Teacher. What should I do next?

Pupils. Add the tens.*

Teacher. 5 and 3 are how many?

Pupils. 8.

Teacher. Where shall I write the 8?

Pupils. Under the tens.*

Teacher. 58 and 31 are how much?

Pupils. 89.

Teacher. (Writes

28	45	57	57	82	64	75	54	25	65
<u>21</u>	<u>23</u>	<u>21</u>	<u>32</u>	<u>12</u>	<u>23</u>	<u>13</u>	<u>22</u>	<u>23</u>	<u>12</u>

on the board.) Mary, you may go to the board and show us how to add the first example.*

Mary. (Goes to board and points.) 8 and 1 are 9. (Writes 9 under ones.) 2 and 2 are 4. (Writes 4 under the tens.)

* If the pupils do not make this suggestion, the teacher should do so.

Teacher. 28 and 21 are how many?

Mary. 49.

Have as many of the other examples worked in the same way as time permits. Add more if necessary, choosing them from the Drill Examples given below.

Drill Examples

<u>45</u>	<u>78</u>	<u>26</u>	<u>47</u>	<u>52</u>	<u>58</u>	<u>57</u>	<u>45</u>	<u>66</u>
<u>23</u>	<u>21</u>	<u>21</u>	<u>32</u>	<u>32</u>	<u>31</u>	<u>21</u>	<u>22</u>	<u>12</u>
<u>65</u>	<u>44</u>	<u>56</u>	<u>86</u>	<u>25</u>	<u>64</u>	<u>47</u>	<u>26</u>	<u>57</u>
<u>12</u>	<u>23</u>	<u>32</u>	<u>12</u>	<u>22</u>	<u>13</u>	<u>21</u>	<u>22</u>	<u>32</u>
<u>74</u>	<u>67</u>	<u>42</u>	<u>54</u>	<u>76</u>	<u>56</u>	<u>78</u>	<u>27</u>	<u>48</u>
<u>13</u>	<u>11</u>	<u>22</u>	<u>23</u>	<u>12</u>	<u>31</u>	<u>11</u>	<u>21</u>	<u>31</u>
<u>55</u>	<u>76</u>	<u>24</u>	<u>46</u>	<u>47</u>	<u>57</u>	<u>46</u>	<u>67</u>	<u>56</u>
<u>32</u>	<u>22</u>	<u>23</u>	<u>21</u>	<u>22</u>	<u>22</u>	<u>22</u>	<u>12</u>	<u>22</u>
<u>58</u>	<u>54</u>	<u>48</u>	<u>77</u>	<u>68</u>	<u>27</u>	<u>46</u>	<u>28</u>	<u>57</u>
<u>21</u>	<u>33</u>	<u>21</u>	<u>12</u>	<u>11</u>	<u>22</u>	<u>32</u>	<u>21</u>	<u>31</u>
<u>54</u>	<u>87</u>	<u>62</u>	<u>74</u>	<u>25</u>	<u>85</u>	<u>75</u>	<u>54</u>	<u>66</u>
<u>32</u>	<u>12</u>	<u>12</u>	<u>23</u>	<u>23</u>	<u>13</u>	<u>12</u>	<u>22</u>	<u>21</u>
<u>56</u>	<u>44</u>	<u>65</u>	<u>68</u>	<u>52</u>	<u>46</u>	<u>74</u>	<u>62</u>	<u>75</u>
<u>21</u>	<u>32</u>	<u>23</u>	<u>21</u>	<u>22</u>	<u>31</u>	<u>12</u>	<u>22</u>	<u>23</u>
<u>47</u>	<u>76</u>	<u>24</u>	<u>45</u>	<u>67</u>	<u>56</u>	<u>87</u>	<u>72</u>	<u>48</u>
<u>32</u>	<u>11</u>	<u>22</u>	<u>32</u>	<u>21</u>	<u>22</u>	<u>11</u>	<u>12</u>	<u>31</u>
<u>55</u>	<u>67</u>	<u>44</u>	<u>64</u>	<u>74</u>	<u>75</u>	<u>64</u>	<u>76</u>	<u>65</u>
<u>32</u>	<u>22</u>	<u>23</u>	<u>12</u>	<u>22</u>	<u>22</u>	<u>22</u>	<u>21</u>	<u>22</u>
<u>85</u>	<u>45</u>	<u>84</u>	<u>77</u>	<u>86</u>	<u>72</u>	<u>64</u>	<u>82</u>	<u>75</u>
<u>12</u>	<u>33</u>	<u>12</u>	<u>21</u>	<u>11</u>	<u>22</u>	<u>23</u>	<u>12</u>	<u>13</u>

FOURTH DAY (20 MINUTES)

Teacher. Yesterday we learned how to add large numbers. How many of you remember how? Suppose we have 47. (Write 47

on board.) Who can show us how to add 21 to it? John, you may show the class.

John. (Goes to board and writes 21, under the 47.) 7 and 1 are 8 (Writes the 8) 4 and 2 are 6. (Writes the 6.)

Teacher. 47 and 21 are how many?

John. 68.

Teacher. That is correct. (Writes on the board 45 56 26 65 57.) Harry may go to the board and show us how to add 23 to 45.

Harry. (Goes to board and writes 23 under 45.) 5 and 3 are 8. (Writes 8.) 4 and 2 are 6. (Writes 6.)

Teacher. 45 and 23 are how many?

Pupils. 68.

Next have different pupils show how to add 31 to 56, 21 to 26, 12 to 65, and 21 to 57. Then draw five large nuts on the board and write the following examples on them.

47	68	66	64	86
<u>32</u>	<u>21</u>	<u>22</u>	<u>23</u>	<u>11</u>

Teacher. Now we are going to play that we are Cracking Nuts.

To crack the nuts you must copy the examples and add. I want to see how many of you can crack all five. Stand when you have all of the nuts cracked.

Supervise this work closely. Be sure that all of the pupils understand the method of procedure. Help any who need assistance. Wait until about two-thirds of the pupils have finished and call for answers. Ask how many cracked all five? How many cracked four? Continue as follows:

Teacher. Write 52 on your paper. Add 32 to it. What is your result, Henry?

Henry. 84.

Teacher. How many have 84?

Dictate the following in the same way:

62	74	48	65
<u>12</u>	<u>12</u>	<u>31</u>	<u>23</u>

Supervise this work carefully. Continue as follows:

Teacher. Harry has a new book that tells about trips in a flying machine. Last night he read 47 pages, and today he read 31 pages. How many pages has he read? Raise your hand when you have the answer. How many pages has Harry read, Louise?

Lowise. 78 pages.

Teacher. You may go to the board and show the class how you found the answer.

Lowise. (Goes to board and writes 47.)

$$\begin{array}{r} 31 \\ \overline{)78} \end{array}$$

Handle the following problems in the same way. Make up more, if necessary, using the numbers given in the Drill Examples (Third Day).

Mary earned 28 cents last week, helping her mother. This week she has earned 21 cents. How much has she earned altogether?

Mr. and Mrs. Squirrel put away 54 nuts for winter. The squirrel children put away 22. How many nuts did they all put away?

Mr. and Mrs. Frog started a singing school. There were 27 froggies in Mr. Frog's class and 22 in Mrs. Frog's. How many froggies were in the singing school?

One morning Harry saw a large flock of wild geese flying north. He counted 65 geese. In the afternoon he saw a smaller flock and counted 13. How many geese did he count?

The first and second grades had a Flag Day drill. There were 44 first grade pupils and 32 second grade pupils. How many children were in the drill?

FIFTH DAY (20 MINUTES)

Teacher. Harry spent 52 cents for a reader and 32 cents for a speller. Find out how much he spent for both. Raise your hand when you have finished. How much did Harry spend?

John. 84 cents.

Teacher. You may show the class what you did.

John. (Goes to board and writes 52.)

$$\begin{array}{r} 32 \\ \overline{)84} \end{array}$$

Play Cracking Nuts. Use the following examples:

52	78	25	45	87
<u>32</u>	<u>21</u>	<u>22</u>	<u>22</u>	<u>12</u>

Teacher. I have put five nuts on the board. Copy and add. I want to see how many of you can crack all five. Some of you

could not crack them all yesterday. Stand when you have finished.

Supervise this work closely. Be sure that all the pupils understand the method of procedure. Help any who need assistance. Wait until about two-thirds of the pupils have finished and call for answers. Ask how many cracked all five. Four? Continue as follows:

Teacher. Write 47 on your paper. Add 32 to it. What is your result, Jane?

Jane. 79.

Teacher. How many have 79?

Dictate the following in the same way:

57	26	42	24
<u>21</u>	<u>22</u>	<u>32</u>	<u>23</u>

Supervise this work closely.

Next divide the pupils into two equal groups and have a Relay Race. If there are an odd number of pupils appoint one as Judge. If there are an even number of pupils the teacher acts as Judge. Choose as many examples from the Drill Examples (Third Day) as there are pupils on each team. Write these on the board in two different places. Line the teams up, one in front of each group of examples. At a given signal the first pupil goes to the board and adds the first example. He then returns to his place in line and hands the chalk to the next pupil on his team. This pupil goes to the board and first makes any corrections he thinks necessary in the first example, then adds the next. Etc. Each pupil, before adding his example, may make corrections in any of the preceding examples. The team that finishes first and has all of its examples right is the winner. If the team that finishes first does not have all of the examples correct it is disqualified. If neither team has all correct it is "no race." The judge declares the winner.

If the pupils have never held such a race before, describe it to them carefully before starting. If the teacher prefers and there are a suitable number of pupils, they may be divided into three teams. In any case the different teams should all be given the same examples.

If time permits a second race may be held.

SIXTH DAY (20 MINUTES)

Teacher. How many of you remember the test I gave you several days ago? Most of you did not work very many of the examples because you did not know how to add and subtract large numbers. We have been practicing on adding large numbers for several days so I am going to give you the same test again and I am sure you can work more examples this time. (Pass Test No. 1.) There are still some examples on the test that are a little different from any we have had but I believe some of you can work them.

Give Test No. 1. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers and proceed as follows:

Teacher. Now I want to see how well you can subtract. We have not been practicing on subtraction for several days but what we have learned about adding large numbers may help you.

Give Test No. 2. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers.

If any time remains, give problems similar to the one given below. Use the numbers given in the Drill Examples (Third Day).

John and his brother Harry sell papers after school. Yesterday John sold 28 and Harry sold 21. How many did they both sell?

SEVENTH DAY (20 MINUTES)

Teacher. Yesterday we had a test on addition and subtraction. Today I am going to give you a different kind of a test. (Pass blanks for Test No. 3.)

Give Test No. 3. Follow exactly the Directions for Administering. Use any time that remains for applied problems similar to those suggested for the Sixth Day.

EIGHTH DAY (20 MINUTES)

Teacher. Alice went to the store for her mother and bought a pound of butter for 45 cents and a half dozen eggs for 22 cents. Find how much she had to pay the clerk. What is your answer, John?

John. 67 cents.

Teacher. You may go to the board and show the class how you got it.

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John. (Goes to board and writes 45.)

$$\begin{array}{r} 22 \\ \underline{67} \end{array}$$

Teacher. Harry went to the store for his mother and bought a cake of soap for 5 cents, a spool of thread for 3 cents, and a stick of chewing gum for himself for 1 cent. Find how much he had to pay the clerk. What is your answer, Tom?

Tom. 9 cents.

Teacher. That is correct. You may go to the board and show the class how you got it.

Tom. (Goes to board and writes 5.)

$$\begin{array}{r} 3 \\ \underline{1} \\ 9 \end{array}$$

Teacher. Mary bought a doll for 32 cents, a doll hat for 22 cents, and a doll dress for 25 cents. What must we do to find out how much she had to pay for all three?

Pupils. Add.

Teacher. That is correct. We have never added three large numbers before so let us see if we can find out how to do it. (Writes 32 on board.) How many tens are there in 32, Mary?

$$\begin{array}{r} 22 \\ 25 \\ \hline \end{array}$$

Mary. Three.

Teacher. How many ones are there?

Mary. Two.

Teacher. So we can write "32 = 3 tens and 2 ones." (Write as you talk.) How many tens and how many ones are there in 22, Louise?

Louise. 2 tens and 2 ones.

Teacher. That is correct. (Write "2 tens and 2 ones" after the 22.) How many tens and how many ones are there in 25, Agnes?

Agnes. 2 tens and 5 ones.

Teacher. (Write "2 tens and 5 ones" after the 25.) Now we are ready to add. How can we find how many ones there will be in the sum, John?

John. Add the ones.

Teacher. (Pointing as you add) 2 and 2 are 4, 4 and 5 are 9.
(Write "9 ones" in proper place.) How can we find how many
tens there will be in the sum?

Pupils. Add the tens.

Teacher. (Pointing as you add.) 3 and 2 are 5, 5 and 2 are 7.
(Writes "7 tens" in proper place.) 7 tens and 9 ones are how
many?

Pupils. 79.

Teacher. (Write 79 below 32.) How much did Mary have to pay

22
25

the clerk for the doll clothes?

Pupils. 79 cents.

Teacher. John stood at the window and counted the people who
passed. He counted 55 men, 23 women, and 21 children. How
can we find out how many people passed?

Pupils. Add.

Teacher. Let us see if we can add this time without writing out the
tens and ones. (Write 55 on the board.) What is the first thing

23

21

to do, Henry?

Henry. Add the ones.

Teacher. You may go to the board and add the ones for us, Jane.

Jane. (Goes to board and writes 9 under the ones.)

Teacher. What is the next thing to do Thomas?

Thomas. Add the tens.

Teacher. You may add the tens for us, Helen.

Helen. (Goes to board and writes 9 under the tens.)

Teacher. How many people did John count?

Pupils. 99.

Teacher. Take pencil and paper. Add 24, 22 and 21. (Dictate
slowly so pupils can write the numbers.) Raise your hand when
you are through. What is your result, Dick?

Dick. 67.

Teacher. How many have 67?

Dictate, in the same way, other examples taken from those given
below. Supervise this work closely. Be sure that all of the pupils

understand the method of procedure. Help any who need assistance.

Drill Examples

22	44	55	25	26	45	26	42	45
22	22	22	23	21	22	21	22	22
<u>23</u>	<u>12</u>	<u>12</u>	<u>21</u>	<u>32</u>	<u>11</u>	<u>22</u>	<u>13</u>	<u>21</u>
62	46	24	26	46	42	24	46	45
22	21	22	22	21	22	22	22	22
<u>13</u>	<u>22</u>	<u>21</u>	<u>31</u>	<u>11</u>	<u>23</u>	<u>22</u>	<u>21</u>	<u>12</u>
52	45	62	25	46	26	44	52	25
22	22	12	22	21	22	23	32	22
<u>13</u>	<u>22</u>	<u>13</u>	<u>21</u>	<u>12</u>	<u>21</u>	<u>12</u>	<u>13</u>	<u>22</u>
54	42	44	56	24	44	45	55	42
23	32	23	21	23	23	23	23	32
<u>12</u>	<u>13</u>	<u>21</u>	<u>11</u>	<u>21</u>	<u>22</u>	<u>11</u>	<u>11</u>	<u>23</u>
26	44	46	56	25	45	56	24	46
21	23	21	21	22	23	22	23	22
<u>21</u>	<u>11</u>	<u>21</u>	<u>12</u>	<u>32</u>	<u>21</u>	<u>11</u>	<u>22</u>	<u>11</u>
54	46	66	46	65	56	22	55	45
23	31	11	31	13	32	22	23	33
<u>21</u>	<u>21</u>	<u>12</u>	<u>12</u>	<u>21</u>	<u>11</u>	<u>23</u>	<u>21</u>	<u>21</u>
44	24	46	65	44	26	52	46	56
33	23	31	13	22	21	32	32	21
<u>12</u>	<u>31</u>	<u>22</u>	<u>11</u>	<u>21</u>	<u>31</u>	<u>12</u>	<u>21</u>	<u>21</u>
56	66	46	66	46	62	54	56	45
21	12	31	12	32	12	22	22	33
<u>22</u>	<u>11</u>	<u>11</u>	<u>21</u>	<u>11</u>	<u>23</u>	<u>11</u>	<u>21</u>	<u>11</u>
54	24	24	25	25	55	54	24	22
23	22	22	22	23	22	23	23	22
<u>11</u>	<u>31</u>	<u>32</u>	<u>31</u>	<u>31</u>	<u>21</u>	<u>22</u>	<u>32</u>	<u>32</u>

NINTH DAY (20 MINUTES)

Teacher. Harry counted his building blocks before he put them away last night. He found that he had 44 white blocks and 32 red blocks. How many blocks has he? Raise your hand when you have the answer. How many blocks does Harry have, Jane?

Jane. 76.

Teacher. You may go to the board and show the class how you found the answer.

Jane. (Goes to board and writes 44.)

32

76

Handle the following problems in the same way.

John went to the circus. He paid 25 cents to get in, 23 cents for candy, and 21 cents for peanuts. How much did he spend?

Mary Jane went to the park to see the animals. She counted 24 monkeys in one cage and 22 in another. How many monkeys did she count?

Mary Jane rode on a pony. That cost 25 cents. Then she took a ride on the elephant too. That cost 23 cents. How much did her rides cost?

The second grade had a party in the woods. Their teacher made sandwiches for the party. There were 26 ham sandwiches, and 21 peanut butter sandwiches. How many sandwiches were there?

John's father took him to the store and bought him some new clothes. His suit cost 24 dollars, his overcoat 23 dollars and the rest of his clothes cost 21 dollars. How much did they all cost?

Harry and his sister Agnes want to go to the country next summer for several weeks. Their board will cost them 48 dollars. To go to the country and come back will cost 21 dollars. How much will it cost them altogether?

Betty's family took an automobile trip one day last summer. They drove 46 miles before they stopped to eat dinner. After dinner they drove 21 miles and then stopped to visit some friends. They then drove 12 miles more before reaching home. How far did they drive?

Tom carries papers after school. He carries papers on three streets. On the first street he delivers 55 papers, on the second street he delivers 23 papers, and on the last street 11 papers. How many papers does Tom deliver?

After working as many of these problems, as time permits, have a Relay Race. Use some examples with two numbers to be added and some with three. Select the examples from the lists of Drill Examples given for the Third and Eighth Days.

TENTH DAY (20 MINUTES)

Teacher. We know more about adding large numbers than we did the last time we took the tests, so I am going to let you take them again today to see if you can work more of the examples correctly.

Give Test No. 1. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers and give Test No. 2. If any time remains give applied problems similar to those suggested for the Ninth Day.

ELEVENTH DAY (20 MINUTES)

Teacher. Yesterday we took the first two of the tests in addition and subtraction. Today I am going to let you take the third test.

Give Test No. 3. Follow exactly the Directions for Administering. Use any time that remains for applied problems similar to those suggested for the Ninth Day.

TWELFTH DAY (20 MINUTES)

Teacher. Alice went to the store for her mother and bought a pound of butter, a dozen eggs and a box of matches. The butter cost 52 cents, the eggs 32 cents, and the matches 3 cents.

How would you find out how much Alice had to pay the clerk?

Pupils. Add.

Teacher. The butter cost 52 cents. (Write 52 on the board.)

The eggs cost 32 cents. You may show me where to write the 32, Tom.

Tom. (Goes to board and writes 32 under 52.)

Teacher. How much did the matches cost?

Pupils. 3 cents.

Teacher. Is that 3 ones or 3 tens, Hazel?

Hazel. 3 ones.

Teacher. Where must we write the 3 ones, Betty?

Betty. Under the ones.

Teacher. You may show us.

Betty. (Goes to board and writes 3 under the ones column.)

Teacher. You may go to the board, Dick, and show us how to add.

Dick. (Goes to board and writes 87 below the column.)

Teacher. Can you tell the class what you did?

Dick. First I added the ones. 2 and 2 are 4, 4 and 3 are 7, so I wrote 7 under the ones. Then I added the tens. 5 and 3 are 8, so I wrote 8 under the tens.

Teacher. How much did Alice have to pay the clerk?

Pupils. 87 cents.

Teacher. Henry counted the number of steps he took in going from his house to the grocery store. He took 45 steps from his house to the corner, 22 from the corner to the door of the grocery store, and 2 from the door to the counter. How many steps did he take altogether? Who can show us how to write the 45 and 22? Louise, you may.

Louise. (Goes to board and writes 45.)

22

Teacher. Where must we write the 2?

Pupils. Under the ones?

Teacher. You may show us, Albert.

Albert. (Goes to board and writes 2 under the ones column.)

Teacher. You may add for us, John.

John. (Goes to board and writes 69 in the proper place.)

Teacher. How many steps did Henry take in going to the grocery store?

Class. 69.

Teacher. Take pencil and paper. Write 62. Write 12 under the 62. Write 3 under the 12. Be sure you write the 3 under the ones. Add. What is your answer, Alice?

Alice. 77.

Teacher. How many have 77? 77 is correct. Now write 46. Write 32 under the 46. Write 1 under the 32. Add. What is your answer Harry?

Harry. 79.

Dictate more examples in the same way. Choose the examples from those given below. Supervise this work closely to make certain all are placing the numbers correctly. Always dictate the numbers from the top down, the two two-place numbers first, and the one place number last.

Drill Examples

44	22	55	25	26	45	26	42	45
22	22	22	23	21	22	21	22	32
2	3	2	1	2	1	2	3	1
62	46	24	42	24	52	44	25	46
22	21	22	32	22	32	21	22	22
3	2	1	3	2	3	2	2	1
62	75	56	76	55	64	62	24	85
12	22	32	11	33	22	32	23	12
3	2	1	2	1	2	3	2	1
72	44	52	54	74	86	25	82	56
12	32	22	32	23	11	22	12	21
2	1	3	2	1	2	1	3	2

THIRTEENTH DAY (20 MINUTES)

Have the following on the blackboard.

		45	52	46
78	85	22	32	21
21	13	11	3	2
25		62	42	86
22	47	12	32	11
32	22	3	23	2

Teacher. There are ten addition examples on the board. Some of them are written correctly and some are not. Who sees one that is wrong? Which one do you think is wrong, Mary?

Mary. The second one.

Teacher. You may go to the board and correct it.

Mary. (Goes to the board and erases the 13 and writes it in proper place 85.)

13

Teacher. Is that right now?

Class. Yes.

Teacher. Who sees another one that is wrong?

Continue this until all have been corrected. Then have a Relay Race. Use a mixed list of examples similar to the ten given above, and chosen from the Drill Examples given for the Third, Eighth, and Twelfth Days. Have some of the examples written with the addends incorrectly placed as in the ten examples given above. Each pupil is first to correct the writing of his example, if necessary, and then add.

After the Relay Race proceed as follows:

Teacher. Last week John's mother gave him a new toy bank.

John put 62 cents in his bank last week. He put 12 cents in yesterday, and 3 cents today. How many cents has he in the bank? Raise your hand when you have the answer. How many cents are in the bank, Mary?

Mary. 77 cents.

Teacher. You may go to the board and show the class how you found the answer.

Mary. (Goes to board and writes 62.)

$$\begin{array}{r} 12 \\ 3 \\ \hline 77 \end{array}$$

Handle the following problems in the same way. Make up more, if necessary, using the numbers given in the Drill Examples, for the Third, Eighth, and Twelfth Day.

Mary's class had a school picnic. There were 26 girls and 21 boys. How many children were at the picnic?

Harry, James and Richard were playing marbles. Harry had 26 marbles, James had 21, and Richard had 31. How many marbles did they have?

Harold lives in the country. One Saturday last summer he set up a fruit stand by the road. He sold 24 baskets of apples, 22 baskets of peaches, and 32 baskets of grapes. How many baskets of fruit did he sell?

John and Agnes are twins. John weighs 42 pounds and Agnes weighs 32 pounds. How many pounds would they weigh standing on the scales together?

Louise went to the grocery for her mother and bought a can of peaches for 25 cents, a can of corn for 22 cents and some candy for herself for 1 cent. How many cents did she pay for all?

Harriet has read 56 pages in her new book. If she reads 32 pages today there will be only 11 pages left. How many pages are there in the book?

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Mary and her mother were playing a number game. Mary's mother said "I am thinking of a number, can you guess what it is?" Mary guessed 52. "Oh, no!", her mother said, "add 32 to that and you will have the right number." What number was Mary's mother thinking of?

Dorothy went to the toy store yesterday and bought some new clothes for her doll. She paid 42 cents for a dress, 22 cents for a pair of shoes and 3 cents for a hat. How much did the clothes cost?

FOURTEENTH DAY (20 MINUTES)

Teacher. We know more about adding large numbers than we did the last time we took the tests, so I am going to let you take them again today to see if you can work more of the examples correctly.

Give Test No. 1. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers and give Test No. 2. If any time remains give applied problems similar to those suggested for the Thirteenth Day.

FIFTEENTH DAY

Teacher. Yesterday we took the first two of the tests on addition and subtraction. Today I am going to let you take the third test.

Give Test No. 3. Follow exactly the Directions for Administering.

GROUP D

FIRST DAY

Teacher. We have been adding and subtracting numbers and today I want to find out how well you can do this work. First I am going to give you a test on addition. (Pass Test No. 1.) There are some examples on this test that are a little different from any you have had but I want to see if any of you can work them.

Give Test No. 1. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers and proceed as follows:

Teacher. Now I want to see how well you can subtract. (Pass Test No. 2.) There are some examples on this test that are a

little different from any you have had but I want to see if any of you can work them.

Give Test No. 2. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers.

SECOND DAY

Teacher. Yesterday we had a test on addition and subtraction. Today I am going to give you a different kind of a test. (Pass the blanks for Test No. 3.) I am not sure that you can work these examples but I want to find out if any of you can.

Give Test No. 3. Follow exactly the Directions for Administering.

THIRD DAY (20 MINUTES)

Teacher. On the tests yesterday and the day before there were some addition examples that you did not know how to work because the numbers were larger than any you have ever added. Today we are going to find out how to add large numbers and some day I will give you the tests again.

Harry had 8 marbles. He bought 7 more. How do we find out how many marbles Harry now has? John you may tell us.

John. Add 8 and 7.

Teacher. How many has he?

John. Fifteen.

Teacher. That is correct. Suppose Harry had 24 marbles and bought 22 more. How could we find out how many marbles Harry now has? Louise may tell us.

Louise. Add 24 and 22.

Teacher. That is correct, but we have never added such large numbers before. Let us see if we can find out how to do it. (Writes 24 on the board.) How many of you remember about

22

ones and tens? That is fine. In adding large numbers we must break them up into ones and tens and add them separately. How many tens are there in 24, Harry?

Harry. Two.

Teacher. How many ones are there?

Harry. Four.

Teacher. That is correct. So we can write "24 = 2 tens and 4 ones." (Writes as she talks.) Louise, how many tens are there in 22?

Louise. Two.

Teacher. How many ones are there?

Louise. Two.

Teacher. So what does 22 equal?

Pupils. 2 tens and 2 ones.

Teacher. (Writes "22 = 2 tens and 2 ones" in proper place.)

Now we are ready to add. 4 ones and 2 ones are how many ones, Agnes?

Agnes. 6 ones.

Teacher. (Writes "6 ones" in proper place.) 2 tens and 2 tens are how many tens, Arthur?

Arthur. 4 tens.

Teacher. (Writes "4 tens" in proper place.) 4 tens and 6 ones are how many?

Pupils. 46.*

Teacher. (Writes 46 below 24.) Harry now has how many

$$\begin{array}{r} 24 \\ 46 \\ \hline \end{array}$$
 marbles?

Pupils. 46.

Teacher. Elizabeth and Margaret sat in an automobile waiting for Mother. Elizabeth counted all the people who passed on one side of the street. There were 47. Margaret counted all the people who passed on the other side. There were 21. How would you find out how many people passed on both sides, John?

John. Add 47 and 21.

Teacher. (Writes 47 on the board.) What do we have to do to big

$$\begin{array}{r} 47 \\ 21 \\ \hline \end{array}$$

numbers so we can add them? Can you tell us, Mary?

Mary. Break them up into tens and ones.†

Teacher. Harry what shall I write after the 47?

Harry. 4 tens and 7 ones.

* The work on the board should now appear as shown below:

$$\begin{array}{l} 24 = 2 \text{ tens and } 4 \text{ ones} \\ 22 = 2 \text{ tens and } 2 \text{ ones} \\ \hline 46 = 4 \text{ tens and } 6 \text{ ones} \end{array}$$

† If the pupils do not make this suggestion, the teacher should do so.

Teacher. (Writes as Harry directs.) Ellen, what shall I write after the 21?

Ellen. 2 tens and 1 one.

Teacher. (Writes as directed.) What should we do next? Can you tell us, Edith?

Edith. See how many ones there are.*

Teacher. You may tell us how to find out, Edith.

Edith. 7 ones and 1 one are 8 ones.

Teacher. (Writes "8 ones" in proper place.) What must we do next?

Pupil. Find out how many tens there are.*

Teacher. Louis you may tell us how to find out.

Louis. 4 tens and 2 tens are 6 tens.

Teacher. (Writes 6 tens in proper place.) How many people passed the automobile?

Pupils. 68.

Teacher. (Writes 58 on the board.) I wonder if we could add
31

these numbers without writing so much on the board? Who can suggest what to do first?

Helen. Add the ones.

Teacher. (Pointing) 8 and 1 are how many?

Pupils. 9.

Teacher. Where should I write the 9?

Pupils. Under the ones.*

Teacher. What should I do next?

Pupils. Add the tens.*

Teacher. 5 and 3 are how many?

Pupils. 8.

Teacher. Where shall I write the 8?

Pupils. Under the tens.*

Teacher. 58 and 31 are how much?

Pupils. 89.

Teacher. Let us see what we have learned about adding large number. What is the first thing to do, Henry?

Henry. Add the ones.*

Teacher. Where do we write the sum of the ones, John?

John. Under the ones.*

* If the pupils do not make this suggestion, the teacher should do so.

Teacher. What do we do next Elizabeth?

Elizabeth. Add the tens.

Teacher. Where do we place the sum of the tens, Harry?

Harry. Under the tens.*

Teacher. (Writes $\begin{array}{r} 28 \\ 21 \\ \hline \end{array}$ $\begin{array}{r} 45 \\ 23 \\ \hline \end{array}$ $\begin{array}{r} 57 \\ 21 \\ \hline \end{array}$ $\begin{array}{r} 57 \\ 32 \\ \hline \end{array}$ $\begin{array}{r} 82 \\ 12 \\ \hline \end{array}$ $\begin{array}{r} 64 \\ 23 \\ \hline \end{array}$ $\begin{array}{r} 75 \\ 13 \\ \hline \end{array}$ $\begin{array}{r} 54 \\ 22 \\ \hline \end{array}$ $\begin{array}{r} 25 \\ 23 \\ \hline \end{array}$ $\begin{array}{r} 65 \\ 12 \\ \hline \end{array}$ on

the board.) Mary, you may go to the board and show us how to add the first example.

Mary. (Goes to board and points.) 8 and 1 are 9. (Writes 9 under the ones.) 2 and 2 are 4. (Writes 4 under the tens.)

Teacher. 28 and 21 are how many?

Mary. 49.

Have as many of the other examples worked in the same way as time permits. Add more if necessary, choosing them from the Drill Examples given below:

Drill Examples

$\begin{array}{r} 45 \\ 23 \\ \hline \end{array}$	$\begin{array}{r} 78 \\ 21 \\ \hline \end{array}$	$\begin{array}{r} 26 \\ 21 \\ \hline \end{array}$	$\begin{array}{r} 47 \\ 32 \\ \hline \end{array}$	$\begin{array}{r} 52 \\ 32 \\ \hline \end{array}$	$\begin{array}{r} 58 \\ 31 \\ \hline \end{array}$	$\begin{array}{r} 57 \\ 21 \\ \hline \end{array}$	$\begin{array}{r} 45 \\ 22 \\ \hline \end{array}$	$\begin{array}{r} 66 \\ 12 \\ \hline \end{array}$
$\begin{array}{r} 65 \\ 12 \\ \hline \end{array}$	$\begin{array}{r} 44 \\ 23 \\ \hline \end{array}$	$\begin{array}{r} 56 \\ 32 \\ \hline \end{array}$	$\begin{array}{r} 86 \\ 12 \\ \hline \end{array}$	$\begin{array}{r} 25 \\ 22 \\ \hline \end{array}$	$\begin{array}{r} 64 \\ 13 \\ \hline \end{array}$	$\begin{array}{r} 47 \\ 21 \\ \hline \end{array}$	$\begin{array}{r} 26 \\ 22 \\ \hline \end{array}$	$\begin{array}{r} 57 \\ 32 \\ \hline \end{array}$
$\begin{array}{r} 74 \\ 13 \\ \hline \end{array}$	$\begin{array}{r} 67 \\ 11 \\ \hline \end{array}$	$\begin{array}{r} 42 \\ 22 \\ \hline \end{array}$	$\begin{array}{r} 54 \\ 23 \\ \hline \end{array}$	$\begin{array}{r} 76 \\ 12 \\ \hline \end{array}$	$\begin{array}{r} 56 \\ 31 \\ \hline \end{array}$	$\begin{array}{r} 78 \\ 11 \\ \hline \end{array}$	$\begin{array}{r} 27 \\ 21 \\ \hline \end{array}$	$\begin{array}{r} 48 \\ 31 \\ \hline \end{array}$
$\begin{array}{r} 55 \\ 32 \\ \hline \end{array}$	$\begin{array}{r} 76 \\ 22 \\ \hline \end{array}$	$\begin{array}{r} 24 \\ 23 \\ \hline \end{array}$	$\begin{array}{r} 46 \\ 21 \\ \hline \end{array}$	$\begin{array}{r} 47 \\ 22 \\ \hline \end{array}$	$\begin{array}{r} 57 \\ 22 \\ \hline \end{array}$	$\begin{array}{r} 46 \\ 22 \\ \hline \end{array}$	$\begin{array}{r} 67 \\ 12 \\ \hline \end{array}$	$\begin{array}{r} 56 \\ 22 \\ \hline \end{array}$
$\begin{array}{r} 58 \\ 21 \\ \hline \end{array}$	$\begin{array}{r} 54 \\ 33 \\ \hline \end{array}$	$\begin{array}{r} 48 \\ 21 \\ \hline \end{array}$	$\begin{array}{r} 77 \\ 12 \\ \hline \end{array}$	$\begin{array}{r} 68 \\ 11 \\ \hline \end{array}$	$\begin{array}{r} 27 \\ 22 \\ \hline \end{array}$	$\begin{array}{r} 46 \\ 32 \\ \hline \end{array}$	$\begin{array}{r} 28 \\ 21 \\ \hline \end{array}$	$\begin{array}{r} 57 \\ 31 \\ \hline \end{array}$
$\begin{array}{r} 54 \\ 32 \\ \hline \end{array}$	$\begin{array}{r} 87 \\ 12 \\ \hline \end{array}$	$\begin{array}{r} 62 \\ 12 \\ \hline \end{array}$	$\begin{array}{r} 74 \\ 23 \\ \hline \end{array}$	$\begin{array}{r} 25 \\ 23 \\ \hline \end{array}$	$\begin{array}{r} 85 \\ 13 \\ \hline \end{array}$	$\begin{array}{r} 75 \\ 12 \\ \hline \end{array}$	$\begin{array}{r} 54 \\ 22 \\ \hline \end{array}$	$\begin{array}{r} 66 \\ 21 \\ \hline \end{array}$

* If the pupils do not make this suggestion, the teacher should do so.

Drill Examples—(Continued.)

56	44	65	68	52	46	74	62	75
<u>21</u>	<u>32</u>	<u>23</u>	<u>21</u>	<u>22</u>	<u>31</u>	<u>12</u>	<u>22</u>	<u>23</u>
47	76	24	45	67	56	87	72	48
<u>32</u>	<u>11</u>	<u>22</u>	<u>32</u>	<u>21</u>	<u>22</u>	<u>11</u>	<u>12</u>	<u>31</u>
55	67	44	64	74	75	64	76	65
<u>32</u>	<u>22</u>	<u>23</u>	<u>12</u>	<u>22</u>	<u>22</u>	<u>22</u>	<u>21</u>	<u>22</u>
85	45	84	77	86	72	64	82	75
<u>12</u>	<u>33</u>	<u>12</u>	<u>21</u>	<u>11</u>	<u>22</u>	<u>23</u>	<u>12</u>	<u>13</u>

End the recitation as follows:

Teacher. Who will tell what we have learned today about adding large numbers? Henry, you may tell us.

Henry. First add the ones and write the sum under the ones.
Then add the tens and write the sum under the tens.

FOURTH DAY (20 MINUTES)

Teacher. Who remembers what we learned yesterday about adding large numbers? Herbert, you may tell the class.

Herbert. First add the ones and write the sum under the ones.
Then add the tens and write the sum under the tens.

Teacher. That is correct. Suppose we have 47 (Writes 47 on board) and want to add 21 to it. Where should we write the 21, John?

John. Under the 47.

Teacher. Is this the way? (Writes 47 .)

21

Pupils. No.

Teacher. Is this the way? (Writes 47.)

21

Pupils. No.

Teacher. Why not, Alice?

Alice. We want to add the ones to the ones and the tens to the tens, so we must write the ones under the ones and the tens under the tens.*

* If the pupils do not make this suggestion, the teacher should do so.

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Teacher. That is fine. You may come to the board, Alice, and show us where to write the 21.

Alice. (Goes to board and writes 47.)

21

Teacher. Who can tell the class everything we know about adding large numbers? Mary, you may tell us.

Mary. First write the numbers with the ones under the ones and the tens under the tens. Then add the ones and write the sum under the ones. Then add the tens and write the sum under the tens.

Teacher. That is correct. (Writes on the board 45 56 26 65 57.) Harry may go to the board and show us how to add 23 to 45.

Harry. (Goes to board and writes 23 under 45.) 5 and 3 are 8 (Writes 8.) 4 and 2 are 6. (Writes 6.)

Teacher. 45 and 23 are how many?

Pupils. 68.

Next have different pupils show how to add 31 to 56, 21 to 26, 12 to 65, and 21 to 57. Then draw five large nuts on the board and write the following examples on them.

47	68	66	64	86
<u>32</u>	<u>21</u>	<u>22</u>	<u>23</u>	<u>11</u>

Teacher. Now we are going to play that we are Cracking Nuts.

To crack the nuts you must copy the examples and add. I want to see how many of you can crack all five. Stand when you have all of the nuts cracked.

Supervise this work closely. Be sure that all of the pupils understand the method of procedure. Help any who need assistance. Wait until about two-thirds of the pupils have finished and call for answers. Ask how many cracked all five? How many cracked four? Continue as follows:

Teacher. Write 52 on your paper. Add 32 to it. What is your result, Henry?

Henry. 84.

Teacher. How many have 84?

Dictate the following in the same way:

62	74	48	65
<u>12</u>	<u>12</u>	<u>31</u>	<u>23</u>

Supervise this work carefully. If any of the pupils have difficulty in placing the numbers, or in adding, call on some other pupil to tell them how to add large numbers. (Ones under ones, etc.) Continue as follows:

Teacher. Harry has a new book that tells about trips in a flying machine. Last night he read 47 pages, and today he read 31 pages. How many pages has he read? Raise your hand when you have the answer. How many pages has Harry read, Louise?

Louise. 78 pages.

Teacher. You may go to the board and show the class how you found the answer.

Louise. (Goes to board and writes 47.)

$$\begin{array}{r} 31 \\ 47 \\ \hline 78 \end{array}$$

Handle the following problems in the same way. Make up more, if necessary, using the numbers given in the Drill Examples. (Third Day.)

Mary earned 28¢ last week, helping her mother. This week she has earned 21¢. How much has she earned altogether?

Mr. and Mrs. Squirrel put away 54 nuts for winter. The squirrel children put away 22. How many nuts did they all put away?

Mr. and Mrs. Frog started a singing school. There were 27 froggies in Mr. Frog's class and 22 in Mrs. Frog's. How many froggies were in the singing school?

One morning Harry saw a large flock of wild geese flying north. He counted 65 geese. In the afternoon he saw a smaller flock and counted 13. How many geese did he count?

The first and second grades had a Flag Day Drill. There were 44 first grade pupils and 32 second grade pupils. How many children were in the drill?

End the recitation by having the pupils tell what they know about adding large numbers. Proceed as follows:

Teacher. How do we write large numbers when we want to add them, John?

John. We write the numbers with the ones under the ones and the tens under the tens.

Teacher. What do we do next, Betty?

Betty. Add the ones.

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Teacher. Where do we put this sum?

Betty. Under the ones.

Etc., Etc.

FIFTH DAY (20 MINUTES)

Teacher. Who remembers what we learned yesterday about adding large numbers? Agnes, you may tell the class.

Agnes. First write the numbers with the ones under the ones and the tens under the tens. Then add the ones and write the sum under the ones. Next add the tens and write the sum under the tens.

Teacher. Harry spent 52 cents for a reader and 32 cents for a speller. Find out how much he spent for both. Raise your hand when you have finished. How much did Harry spend?

John. 84 cents.

Teacher. You may show the class what you did.

John. (Goes to board and writes 52.)

$$\begin{array}{r} 32 \\ \hline 84 \end{array}$$

Play Cracking Nuts. Use the following examples:

$$\begin{array}{r} 52 \\ \hline 32 \end{array} \quad \begin{array}{r} 78 \\ \hline 21 \end{array} \quad \begin{array}{r} 25 \\ \hline 22 \end{array} \quad \begin{array}{r} 45 \\ \hline 22 \end{array} \quad \begin{array}{r} 87 \\ \hline 12 \end{array}$$

Teacher. I have put five nuts on the board. Copy and add. I want to see how many of you can crack all five. Some of you could not crack them all yesterday. Stand when you have finished.

Supervise this work closely. Be sure that all the pupils understand the method of procedure. Help any who need assistance. Wait until about two-thirds of the pupils have finished and call for answers. Ask how many cracked all five. Continue as follows:

Teacher. Write 47 on your paper. Add 32 to it. What is your result, Jane?

Jane. 79.

Teacher. How many have 79?

Dictate the following in the same way:

$$\begin{array}{r} 57 \\ \hline 21 \end{array} \quad \begin{array}{r} 26 \\ \hline 22 \end{array} \quad \begin{array}{r} 42 \\ \hline 32 \end{array} \quad \begin{array}{r} 24 \\ \hline 23 \end{array}$$

Supervise this work carefully. If any of the pupils have difficulty in placing the numbers, or in adding, call on some pupil to tell them how to add large numbers. (Ones under ones, etc.)

Next divide the pupils into two equal groups and have a Relay Race. If there are an odd number of pupils appoint one as Judge. If there are an even number of pupils the teacher acts as Judge. Choose as many examples from the Drill Examples (Third Day) as there are pupils on each team. Write these on the board in two different places. Line the teams up, one in front of each group of examples. At a given signal the first pupil goes to the board and adds the first example. He then returns to his place in line and hands the chalk to the next pupil on his team. This pupil goes to the board and first makes any corrections he thinks necessary in the first example, then adds the next. Etc. Each pupil, before adding his example, may make corrections on any of the preceeding examples. The team that finishes first and has all of its examples right is the winner. If the team that finishes first does not have all of the examples correct it is disqualified. If neither team has all correct it is "no race." The judge declares the winner.

If the pupils have never held such a race before, describe it to them carefully before starting. If the teacher prefers and there are a suitable number of pupils, they may be divided into three teams. In any case the different teams should all be given the same examples.

If time permits a second race may be held. End the recitation by having the pupils summarize what they know about adding large numbers.

SIXTH DAY (20 MINUTES)

Teacher. How many of you remember the test I gave you several days ago? Most of you did not work very many of the examples because you did not know how to add and subtract large numbers. We have been practicing on adding large numbers for several days so I am going to give you the same test again and I am sure you can work more examples this time. (Pass Test No. 1.) There are still some examples on the test that are a little different from any we have had but I believe some of you can work them.

Give Test No. 1. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers and proceed as follows:

Teacher. Now I want to see how well you can subtract. We have not been practicing on subtraction for several days but what we have learned about adding large numbers may help you.

Give Test No. 2. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers.

If any time remains, give problems similar to the one given below. Use the numbers given in the Drill Examples. (Third Day.)

John and his brother Harry sell papers after school. Yesterday John sold 28 and Harry sold 21. How many did they both sell?

SEVENTH DAY (20 MINUTES)

Teacher. Yesterday we had a test on addition and subtraction.

Today I am going to give you a different kind of a test. (Pass blanks for Test No. 3.)

Give Test No. 3. Follow exactly the Directions for Administering. Use any time that remains for applied problems similar to those suggested for the Sixth Day.

EIGHTH DAY (20 MINUTES)

Teacher. How many can tell us how to add large numbers?

Helen, you may tell the class.

Helen. Write the ones under the ones and the tens under the tens.

Add the ones and write the sum under the ones. Add the tens and write the sum under the tens.

Teacher. Alice went to the store for her mother and bought a pound of butter for 45 cents and a half dozen eggs for 22 cents.

Find how much she had to pay the clerk. What is your answer, John?

John. 67 cents.

Teacher. You may go to the board and show the class how you got it.

John. (Goes to board and writes 45.)

$$\begin{array}{r} 22 \\ \hline 67 \end{array}$$

Teacher. Harry went to the store for his mother and bought a cake of soap for 5 cents, a spool of thread for 3 cents, and a stick

of chewing gum for himself for 1 cent. Find out how much he had to pay the clerk. What is your answer, Tom?

Tom. 9 cents.

Teacher. That is correct. You may go to the board and show the class how you got it.

Tom. (Goes to board and writes 5.)

$$\begin{array}{r} 3 \\ 1 \\ \hline 9 \end{array}$$

Teacher. If you have three numbers to add how do you write them?

Pupils. In a column, the second number under the first, and the third under the second.*

Teacher. Mary bought a doll for 32 cents, a doll hat for 22 cents, and a doll dress for 25 cents. What must we do to find how much she had to pay for all three?

Pupils. Add.

Teacher. If we had just the 32 and 22 to add, how would we write them, James?

James. Write the 22 under the 32, putting the ones under the ones and the tens under the tens.

Teacher. You may come to the board and show us.

James. (Goes to board and writes 32.)

$$22$$

Teacher. But Mary also bought a doll dress for 25 cents so we must add the 25 also. Where do you think we ought to write it, Elizabeth?

Elizabeth. Under the 22, putting the ones under the ones, and the tens under the tens.*

Teacher. Why must we write the ones under the ones and the tens under the tens?

Elizabeth. So we can add the ones to the ones, and the tens to the tens.*

Teacher. You may show us, Elizabeth, where to write the 25.

Elizabeth. (Goes to board and writes 25 under the 22.)

Teacher. Now we are ready to add. What must we do first, Joseph?

* If the pupils do not make this suggestion, the teachers should do so.

Joseph. Add the ones.

Teacher. What is the sum of the ones?

Pupils. 9.

Teacher. Where do we write the 9?

Pupils. Under the ones.

Teacher. What must we do next?

Pupils. Add the tens.

Teacher. What is the sum of the tens?

Pupils. 7.

Teacher. Where do we write the 7?

Pupils. Under the tens.

Teacher. How much did Mary have to pay?

Pupils. 79 cents.

Teacher. John stood at the window and counted the people who passed. He counted 55 men, 23 women, and 21 children. Who can show me how to find out how many people passed. Ruth you may show us.

Ruth. (Goes to board and writes 55.)

23

21

99

Teacher. How many people passed?

Ruth. 99.

Teacher. How many can tell us how to add when we have three large numbers? Jack, may tell us.

Jack. Write the second number under the first and the third under the second, keeping ones under ones and tens under tens. Add the ones and write the sum under the ones. Add the tens and write the sum under the tens.

Teacher. Take pencil and paper. Add 24, 22 and 21. (Dictate slowly so pupils can write the numbers in proper place.) Raise your hand when you are through. What is your result, Dick?

Dick. 67.

Teacher. How many have 67?

Dictate in the same way, other examples taken from those given below. Supervise this work closely. Be sure that all of the pupils understand the method of procedure. Help any who need assistance.

Drill Examples

22	44	55	25	26	45	26	42	45
22	22	22	23	21	22	21	22	22
<u>23</u>	<u>12</u>	<u>12</u>	<u>21</u>	<u>32</u>	<u>11</u>	<u>22</u>	<u>13</u>	<u>21</u>
62	46	24	26	46	42	24	46	45
22	21	22	22	21	22	22	22	22
<u>13</u>	<u>22</u>	<u>21</u>	<u>31</u>	<u>11</u>	<u>23</u>	<u>22</u>	<u>21</u>	<u>12</u>
52	45	62	25	46	26	44	52	25
22	22	12	22	21	22	23	32	22
<u>13</u>	<u>22</u>	<u>13</u>	<u>21</u>	<u>12</u>	<u>21</u>	<u>12</u>	<u>13</u>	<u>22</u>
54	42	44	56	24	44	45	55	42
23	32	23	21	23	23	23	23	32
<u>12</u>	<u>13</u>	<u>21</u>	<u>11</u>	<u>21</u>	<u>22</u>	<u>11</u>	<u>11</u>	<u>23</u>
26	44	46	56	25	45	56	24	46
21	23	21	21	22	23	22	23	22
<u>21</u>	<u>11</u>	<u>21</u>	<u>12</u>	<u>32</u>	<u>21</u>	<u>11</u>	<u>22</u>	<u>11</u>
54	46	66	46	65	56	22	55	45
23	31	11	31	13	32	22	23	33
<u>21</u>	<u>21</u>	<u>12</u>	<u>12</u>	<u>21</u>	<u>11</u>	<u>23</u>	<u>21</u>	<u>21</u>
44	24	46	65	44	26	52	46	56
33	23	31	13	22	21	32	32	21
<u>12</u>	<u>31</u>	<u>22</u>	<u>11</u>	<u>21</u>	<u>31</u>	<u>12</u>	<u>21</u>	<u>21</u>
56	66	46	66	46	62	54	56	45
21	12	31	12	32	12	22	22	33
<u>22</u>	<u>11</u>	<u>11</u>	<u>21</u>	<u>11</u>	<u>23</u>	<u>11</u>	<u>21</u>	<u>11</u>
54	24	24	25	25	55	54	24	22
23	22	22	22	23	22	23	23	22
<u>11</u>	<u>31</u>	<u>32</u>	<u>31</u>	<u>31</u>	<u>21</u>	<u>22</u>	<u>32</u>	<u>32</u>

End recitation by having the pupils summarize what they know about adding three large numbers.

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NINTH DAY (20 MINUTES)

Teacher. How many can tell us what we have learned about adding two large numbers? Joseph, you may tell us.

Joseph. Write the second under the first, ones under ones and tens under tens. Add the ones and write the sum under the ones. Add the tens and write the sum under the tens.

Teacher. That is fine. How many can tell us what we have learned about adding *three* large numbers? Hazel may tell us.

Hazel. Write the second under the first, the third under the second, writing ones under ones and tens under tens. Etc.

Teacher. Harry counted his building blocks before he put them away last night. He found that he had 44 white blocks and 32 red blocks. How many blocks has he? Raise your hand when you have the answer. How many blocks does Harry have, Jane?

Jane. 76.

Teacher. You may go to the board and show the class how you found the answer.

Jane. (Goes to board and writes 44.)

$$\begin{array}{r} 32 \\ 44 \\ \hline 76 \end{array}$$

Handle the following problems in the same way.

John went to the circus. He paid 25 cents to get in, 23 cents for candy, and 21 cents for peanuts. How much did he spend?

Mary Jane went to the park to see the animals. She counted 24 monkeys in one cage and 22 in another. How many monkeys did she count?

Mary Jane rode on a pony. That cost 25 cents. Then she took a ride on the elephant too. That cost 23 cents. How much did her rides cost?

The second grade had a party in the woods. Their teacher made sandwiches for the party. There were 26 ham sandwiches, 22 cheese sandwiches, and 21 peanut butter sandwiches. How many sandwiches were there?

John's father took him to the store and bought him some new clothes. His suit cost 24 dollars, his overcoat 23 dollars and the rest of his clothes cost 21 dollars. How much did they all cost?

Harry and his sister Agnes want to go to the country next summer for several weeks. Their board will cost them 48 dollars.

To go to the country and come back will cost 21 dollars. How much will it cost them altogether?

Betty's family took an automobile trip one day last summer. They drove 46 miles before they stopped to eat dinner. After dinner they drove 21 miles and then stopped to visit some friends. They then drove 12 miles more before reaching home. How far did they drive?

Tom carries papers after school. He carries papers on three streets. On the first street he delivers 55 papers, on the second street he delivers 23 papers, and on the last street 11 papers. How many papers does Tom deliver?

After working as many of these problems, as time permits, have a Relay Race. Use some examples with two numbers to be added and some with three. Select the examples from the lists of Drill Examples given for the Third and Eighth Days.

End the recitation by having the pupils summarize what they know about adding large numbers.

TENTH DAY (20 MINUTES)

Teacher. We know more about adding large numbers than we did the last time we took the tests, so I am going to let you take them again today to see if you can work more of the examples correctly.

Give Test No. 1. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers and give Test No. 2. If any time remains give applied problems similar to those suggested for the Ninth Day.

ELEVENTH DAY (20 MINUTES)

Teacher. Yesterday we took the first two of the tests on addition and subtraction. Today I am going to let you take the third test.

Give Test No. 3. Follow exactly the Directions for Administering. Use any time that remains for applied problems similar to those suggested for the Ninth Day.

TWELFTH DAY (20 MINUTES)

Teacher. Alice went to the store for her mother and bought a pound of butter, a dozen eggs and a box of matches. The

butter cost 52 cents, the eggs 32 cents, and the matches 3 cents.

How would you find out how much Alice had to pay the clerk?

Pupils. Add.

Teacher. The butter cost 52 cents. (Writes 52 on the board.)

The eggs cost 32 cents. Where should we write the 32 cents?

Pupils. Under the 52.

Teacher. Is this correct? (Writes 52.)

$$\begin{array}{r} 32 \\ \hline \end{array}$$

Pupils. No.

Teacher. Is this right? (Writes 52.)

$$\begin{array}{r} 32 \\ \hline \end{array}$$

Pupils. No.

Teacher. Well who can show me where to write it? Jane you may.

Jane. (Goes to board and writes 52.)

$$\begin{array}{r} 32 \\ \hline \end{array}$$

Teacher. Is that correct?

Pupils. Yes.

Teacher. Why, Harry?

Harry. Because we must write the ones under the ones and the tens under the tens.

Teacher. How much did the matches cost?

Pupils. 3 cents.

Teacher. Is that 3 ones or 3 tens, Hazel?

Hazel. 3 ones.

Teacher. Where must we write the 3 ones, Betty?

Betty. Under the ones.

Teacher. You may show us.

Betty. (Goes to board and writes 3 under the ones column.)

Teacher. You may go to the board, Dick and show us how to add.

Dick. (Goes to board and writes 87 below the column.)

Teacher. Can you tell the class what you did?

Dick. First I added the ones, 2 and 2 are 4, 4 and 3 are 7, so I wrote 7 under the ones. Then I added the tens. 5 and 3 are 8 so I wrote 8 under the tens.

Teacher. How much did Alice have to pay the clerk?

Pupils. 87 cents.

Teacher. Henry counted the number of steps he took in going from his house to the grocery store. He took 45 steps from his house to the corner, 22 from the corner to the door of the

grocery store, and 2 from the door to the counter. How many steps did he take altogether? Who can show us how to write the 45 and 22? Louise, you may.

Louise. (Goes to board and writes 45.)

22

Teacher. Where must we write the 2?

Pupils. Under the ones.

Teacher. You may show us, Albert.

Albert. (Goes to board and writes 2 under the ones column.)

Teacher. You may add for us, John.

John. (Goes to board and writes 69 in the proper place.)

Teacher. How many steps did Henry take in going to the grocery store?

Class. 69.

Teacher. In addition we must write the ones under the ones, and the tens under the tens. Let us look at the two examples we have worked and see if we can find an easy way of doing this. Which column must we keep straight, the left (Trace down the left column of each example with a pointer), or the right (Trace down the right column.)?

Pupils. The right column.

Teacher. In writing numbers to add we must be careful to keep which column straight?

Pupils. The right column.

Teacher. Why must we keep the right column straight?

Pupils. So the ones will be under the ones and the tens under the tens.

Teacher. Take pencil and paper. Write 62. Write 12 under the 62, being careful to keep the ones under the ones. Write 3 under the 12. Keep ones under the ones. Which column should be straight, Alice?

Alice. The right-hand column.

Teacher. All notice and see if you have the right-hand column straight. Add. What is your answer, Louise?

Louise. 77.

Teacher. How many have 77? 77 is correct. Now write 46. Write 32 under the 46 in the proper place. Write 1 under the 32 in the proper place. Add. What is your answer, Harry?

Harry. 79.

Dictate more examples in the same way. Choose the examples from those given below. Supervise this work closely to make

certain all are placing the numbers correctly. Constantly emphasize the fact that the numbers to be added must be written with the ones under the ones, or so that the right-hand column will be straight. Always dictate the numbers from the top down, the two two-place numbers first, and the one-place number last.

Drill Examples

44	22	55	25	26	45	26	42	45
22	22	22	23	21	22	21	22	32
<u>2</u>	<u>3</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>1</u>
62	46	24	42	24	52	44	25	46
22	21	22	32	22	32	21	22	22
<u>3</u>	<u>2</u>	<u>1</u>	<u>3</u>	<u>2</u>	<u>3</u>	<u>2</u>	<u>2</u>	<u>1</u>
62	75	56	76	55	64	62	24	85
12	22	32	11	33	22	32	23	12
<u>3</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>2</u>	<u>1</u>
72	44	52	54	74	86	25	82	56
12	32	22	32	23	11	22	12	21
<u>2</u>	<u>1</u>	<u>3</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>3</u>	<u>2</u>

THIRTEENTH DAY (20 MINUTES)

Have the following on the blackboard.

		45	52	46
78	85	22	32	21
<u>21</u>	<u>13</u>	<u>11</u>	<u>3</u>	<u>2</u>
25		62	42	86
22	47	12	32	11
<u>32</u>	<u>22</u>	<u>3</u>	<u>23</u>	<u>2</u>

Teacher. How many can tell us what we learned yesterday about writing numbers in addition. Jane you may tell the class.

Jane. You must write them so the right-hand column will be straight.

Teacher. Why must the right-hand column be kept straight?

Jane. So the ones will be under the ones, and the tens under the tens.

Teacher. There are ten addition examples on the board. Some of them are written correctly and some are not. Who sees one that is wrong? Which one do you think is wrong, Mary?

Mary. The second one.

Teacher. You may go to the board and correct it.

Mary. (Goes to the board and erases the 13 and writes it in proper place 85.)

13

Teacher. Is that right now?

Pupils. Yes.

Teacher. Who sees another one that is wrong?

Continue this until all have been corrected. Then have a Relay Race. Use a mixed list of examples similar to the ten given above, and chosen from the Drill Examples given for the Third, Eighth, and Twelfth Days. Have some of the examples written with the addends incorrectly placed as in the ten examples given above. Each pupil is first to correct the writing of his example, if necessary, and then add.

After the Relay Race proceed as follows:

Teacher. Last week John's mother gave him a new toy bank.

John put 62 cents in his bank last week. He put 12 cents in yesterday, and 3 cents today. How many cents has he in the bank? Raise your hand when you have the answer. How many cents are in the bank, Mary?

Mary. 77 cents.

Teacher. You may go to the board and show the class how you found the answer.

Mary. (Goes to board and writes 62.)

$$\begin{array}{r} 12 \\ 3 \\ \hline 77 \end{array}$$

Handle the following problems in the same way. Make up more, if necessary, using the numbers given in the Drill Examples, for the Third, Eighth, and Twelfth Days.

Early in the morning a street car carried people to the city to work. Twenty-four got on where the car started. Twenty-three more got on at the next stop, and two more at the next stop. How many were on the car?

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Mary's class had a school picnic. There were 26 girls and 21 boys. How many children were at the picnic?

Harry, James and Richard were playing marbles. Harry had 26 marbles, James had 21, and Richard had 31. How many marbles did they have?

Harold lives in the country.. One Saturday last summer he set up a fruit stand by the road. He sold 24 baskets of apples, 22 baskets of peaches, and 32 baskets of grapes. How many baskets of fruit did he sell?

John and Agnes are twins. John weighs 42 pounds and Agnes weighs 32 pounds. How many pounds would they weigh standing on the scales together?

Louise went to the grocery for her mother and bought a can of peaches for 25 cents, a can of corn for 22 cents and some candy for herself for 1 cent. How many cents did she pay for all?

Harriet has read 56 pages in her new book. If she reads 32 pages today there will be only 11 pages left. How many pages are there in the book?

Mary and her mother were playing a number game. Mary's mother said "I am thinking of a number, can you guess what it is?" Mary guessed 52. "Oh, no!", her mother said, "Add 32 to that and you will have the right number." What number was Mary's mother thinking of?

Dorothy went to the toy store yesterday and bought some new clothes for her doll. She paid 42 cents for a dress, 22 cents for a pair of shoes and 3 cents for a hat. How much did the clothes cost?

End the recitation as follows:

Teacher. What must we remember in writing numbers in addition? Henry?

Henry. Keep the right-hand column straight.

Teacher. Why must we keep the right-hand column straight? Betty?

Betty. So the ones will be under the ones and the tens under the tens.

Teacher. After we have the numbers written properly what do we do next, Florence?

Etc., Etc.

FOURTEENTH DAY (20 MINUTES)

Teacher. We know more about adding large numbers than we did the last time we took the tests, so I am going to let you take

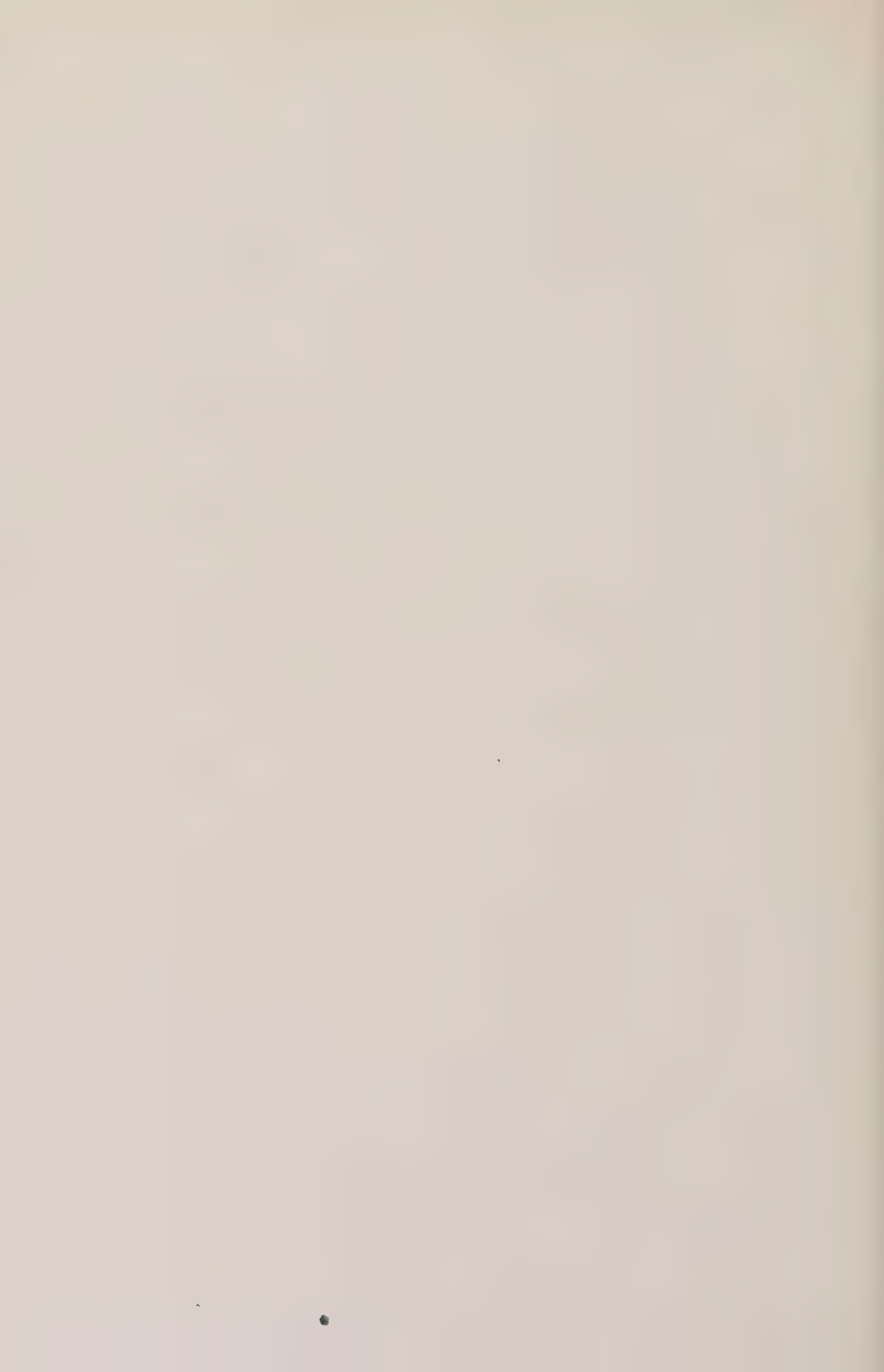
them again today to see if you can work more of the examples correctly.

Give Test No. 1. Follow exactly the Directions for Administering. When you are convinced the pupils have worked all they are able to, take up the papers and give Test No. 2. If any time remains give applied problems similar to those suggested for the Thirteenth Day.

FIFTEENTH DAY

Teacher. Yesterday we took the first two of the tests on addition and subtraction. Today I am going to let you take the third test.

Give Test No. 3. Follow exactly the Directions for Administering.



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